清華大學
Tsinghua University

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Signatures of Cosmic Reionization on the 21cm 2 & 3-Point Correlation

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collaborators:

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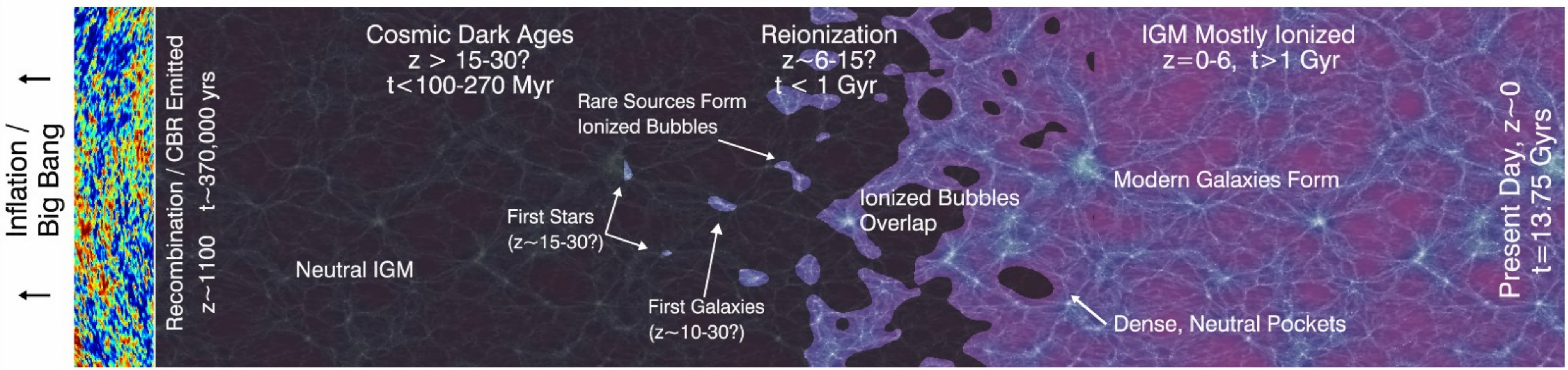
overview

- **Introduction**
- Global neutral fraction
- Correlation functions
- Bias modeling
- Summary

Epoch of Reionization

time



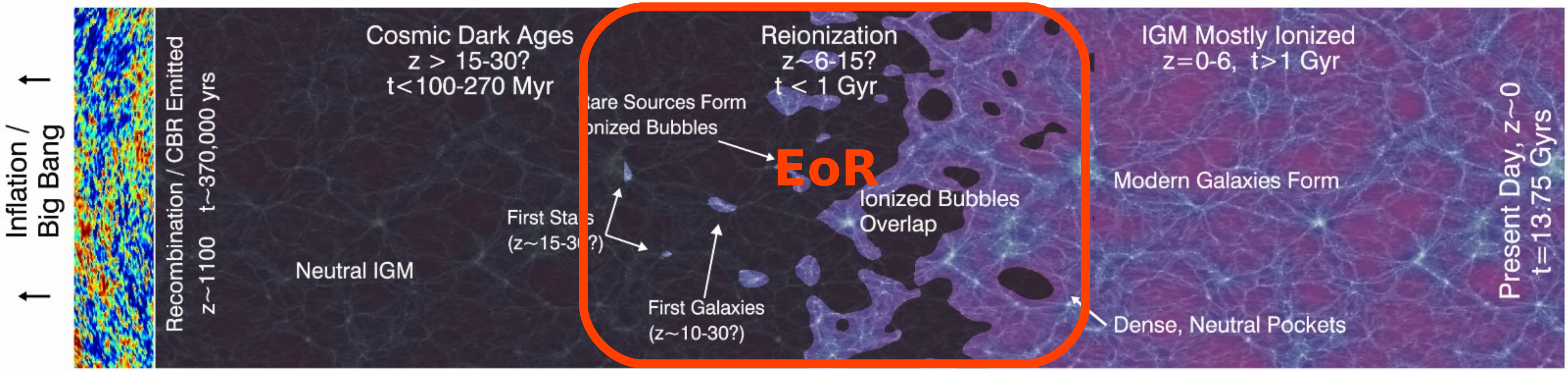
Robertson et al. 2010, Nature, 468, 49-55

Planck, WMAP, ..

BOSS, DES, Euclid,...

Epoch of Reionization

time



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Epoch of Reionization

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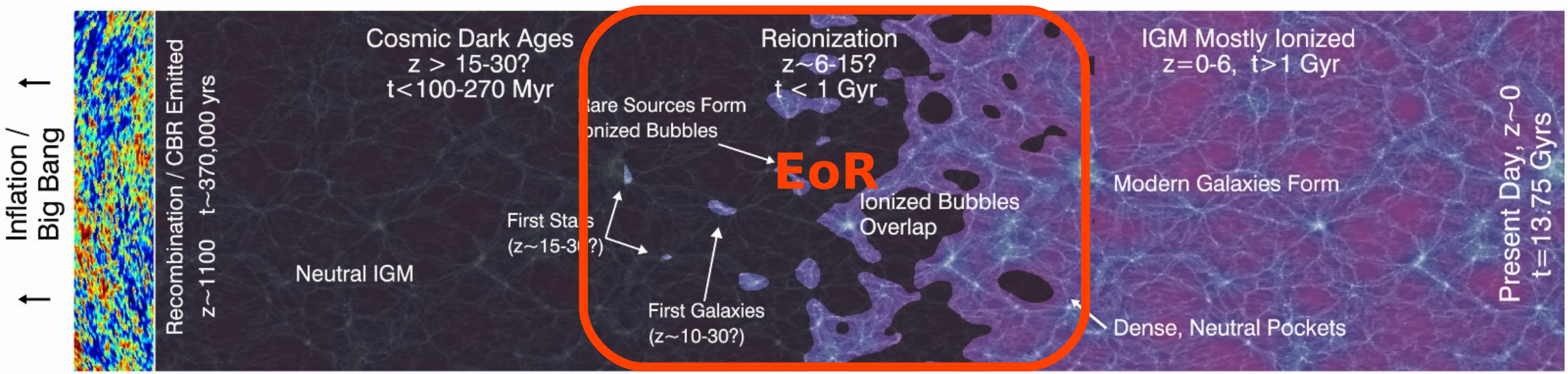

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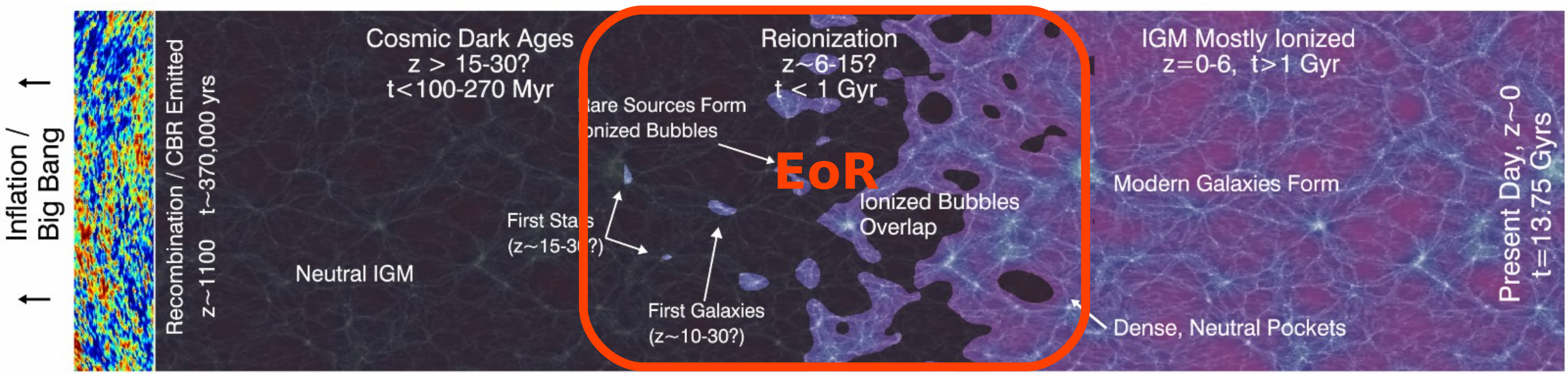
Planck, WMAP, ..

HERA, SKA-low, ..

BOSS, DES, Euclid,...

Epoch of Reionization

time



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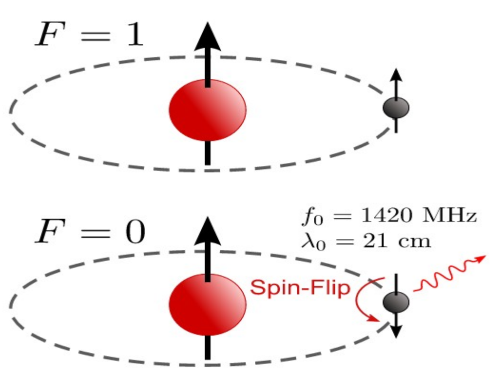
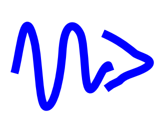
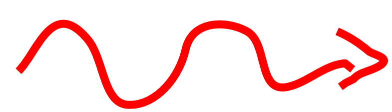


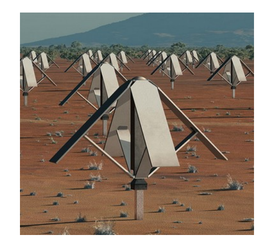
electron spin flip in
neutral hydrogen



21cm

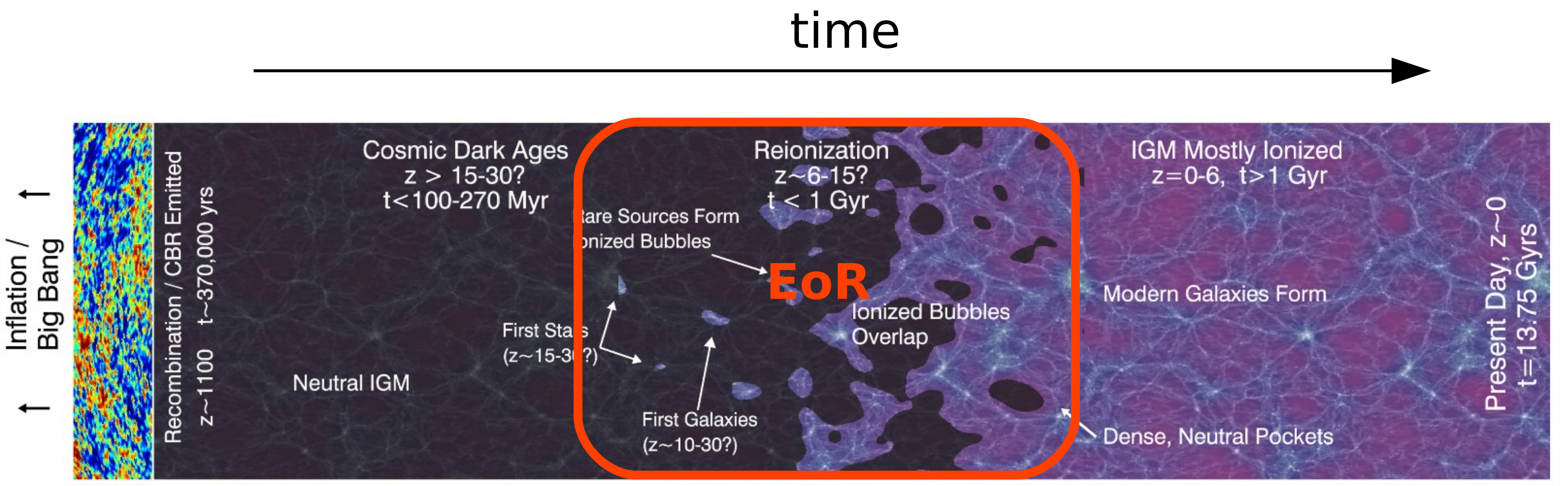


radio



SKA-LOW

Epoch of Reionization



Robertson et al. 2010, Nature, 468, 49-55

morphology of ionized regions depends on:

- how first luminous objects form
- how their radiation transfers energy to the IGM

clustering of the 21cm signal will provide constraints on reionization models

21cm brightness temperature

$$\delta T = \rho_{HI}\left(\frac{T_S - T_{CMB}}{T_S}\right) \times constant$$

21cm brightness temperature

local density of neutral hydrogen

$$\delta T = \rho_{HI} \left(\frac{T_S - T_{CMB}}{T_S} \right) \times constant$$

21cm brightness temperature

local density of neutral hydrogen

hydrogen spin temperature

CMB temperature

$$\delta T = \rho_{HI} \left(\frac{T_S - T_{CMB}}{T_S} \right) \times constant$$

$$23.88 \left(\frac{0.15}{\Omega_M \ h^2} \frac{1+z}{10} \right)^{1/2} \left(\frac{\Omega_b h^2}{0.02} \right) \mathrm{mK}$$

21cm brightness temperature

local density of neutral hydrogen

hydrogen spin temperature

CMB temperature

$$\delta T = \rho_{HI} \left(\underbrace{\frac{T_S - T_{CMB}}{T_S}}_{\substack{= 1 \\ (T_s \gg T_{CMB})}} \right) \times constant$$

21cm brightness temperature

local density of neutral hydrogen

$$\delta T = \rho_{HI} \times constant$$

$$\rho_{HI} = x_{HI} \rho_H$$

21cm brightness temperature

local density of neutral hydrogen

$$\delta T = \rho_{HI} \times constant$$

$$\rho_{HI} = x_{HI} \rho_H \propto x_{HI} \rho_m$$

21cm brightness temperature

local density of neutral hydrogen

$$\delta T = \rho_{HI} \times constant$$

$$\rho_{HI} = x_{HI}\rho_H \propto x_{HI}\rho_m = x_{HI}\langle\rho_m\rangle(1+\delta_m)$$

21cm brightness temperature

$$\delta T \propto x_{HI}(1+\delta_m)$$

simulations of 21cm signal require:

- matter fluctuations: δ_m
- neutral fraction: x_{HI}

21cm brightness temperature

$$\delta T \propto x_{HI}(1+\delta_m)$$

simulations of 21cm signal require:
- matter fluctuations: δ_m
- neutral fraction: x_{HI}

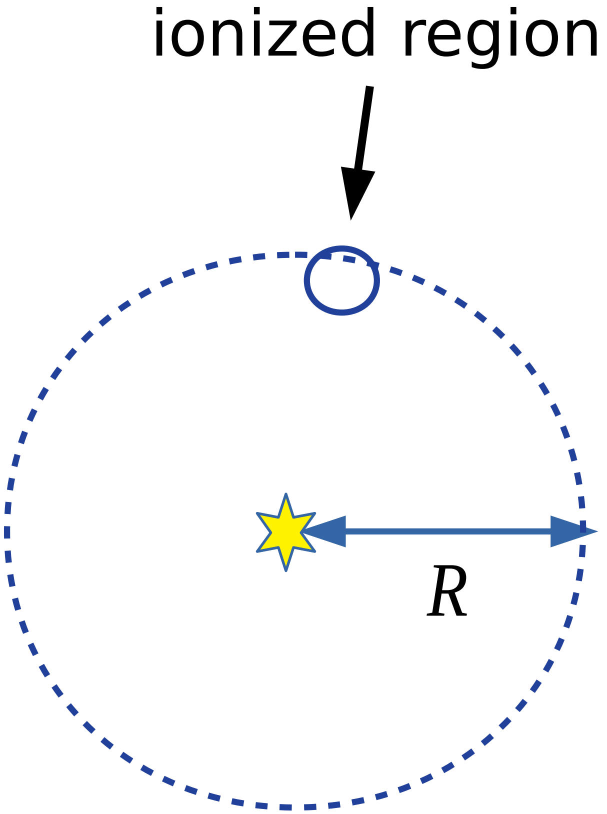
we ignore:
- CMB contributions
- redshift space distortions
- foreground contamination (radio stations, planes, satellites, ionosphere, galactic synchrotron radiation)
-

21cmFAST simulation method

<u>conditions for ionizing source</u>

i) $R < mean\,free\,path\,length\,(mfp)$

ii) $m_{source} > m_{min}$



(21cmFAST: Mesinger et al. 2010)

21cmFAST simulation method

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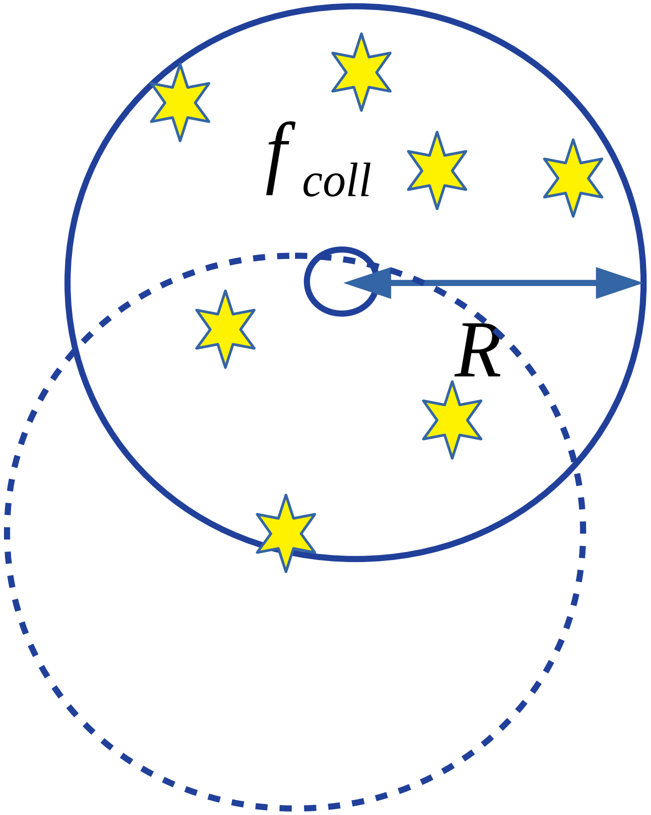


<u>condition for ionized region</u>

$$f_{coll} = erfc\left(\frac{\delta_{crit} - \delta_R}{[2(\sigma^2(m_{min}) - \sigma_R^2)]^{1/2}}\right) > \frac{1}{\zeta}$$

(21cmFAST: Mesinger et al. 2010)

21cmFAST parameters

condition for ionization

$$f_{coll} = erfc\left(\frac{\delta_{crit} - \delta_R}{[2(\sigma^2(m_{min}) - \sigma_R^2)]^{1/2}}\right) > \frac{1}{\zeta}$$

free model parameters

i) mean free path length for ionizing photons R_{max}

(21cmFAST: Mesinger et al. 2010)

21cmFAST parameters

condition for ionization

$$f_{coll} = erfc\left(\frac{\delta_{crit} - \delta_R}{[2(\sigma^2(m_{min}) - \sigma_R^2)]^{1/2}}\right) > \frac{1}{\zeta}$$

free model parameters

i) mean free path length for ionizing photons R_{max}

ii) ionization efficiency ζ depends on

- number of ionizing photons produced per baryon in stars
- escape fraction of ionizing photons from source
- star formation efficiency
- hydrogen recombination rate

21cmFAST parameters

condition for ionization

$$f_{coll} = erfc\left(\frac{\delta_{crit} - \delta_R}{[2(\sigma^2(m_{min}) - \sigma_R^2)]^{1/2}}\right) > \frac{1}{\zeta}$$

free model parameters

i) mean free path length for ionizing photons R_{max}

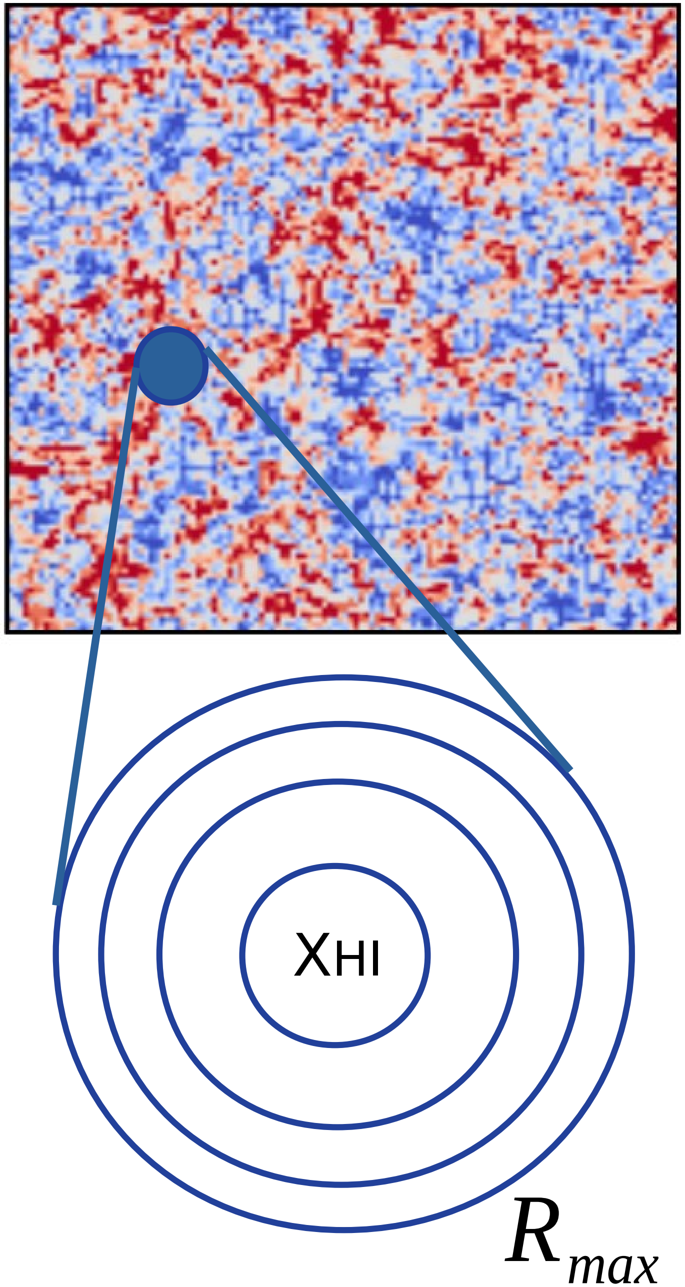
ii) ionization efficiency ζ

iii) minimum mass of ionizing sources ($m_{min} \propto T_{min}^{3/2}$)

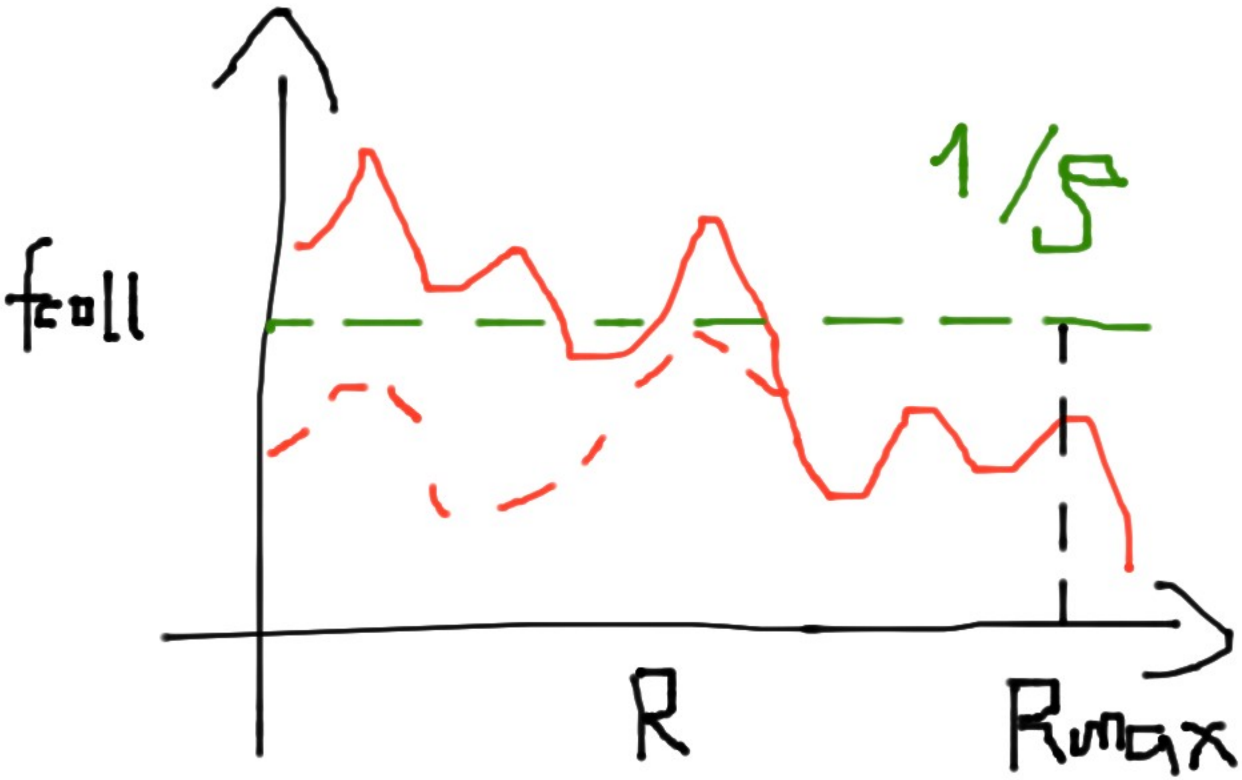
21cmFAST simulation method

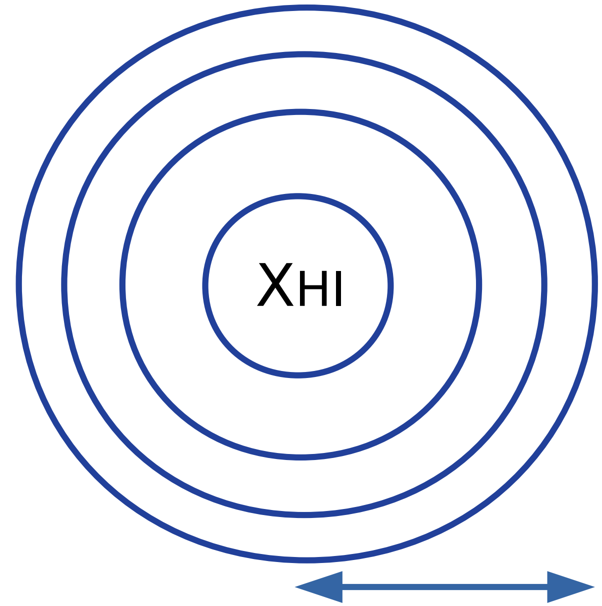
I) run dark matter simulation
(using Zel'dovich approx.)

ii) get neutral fraction x_{HI}
in grid cells from
excursion set model



neutral fraction X_{HI} from excursion set model





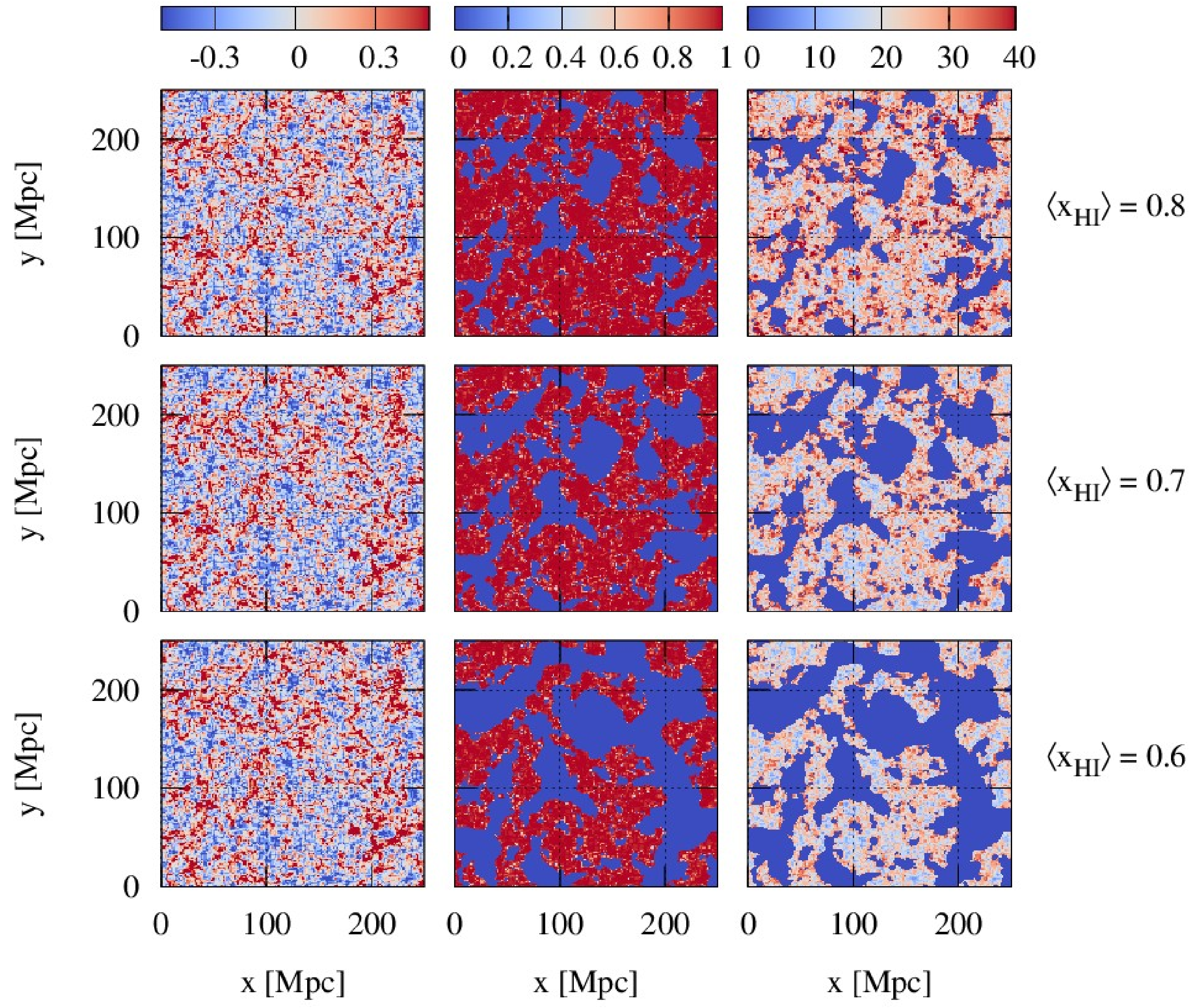
R_{max} = mean free path length

$$\textbf{ionized fraction: } X_{HII} = \begin{cases} 1 \text{ if } \; f_{coll}(R<R_{max})>1/\zeta \\ 0 \text{ else} \end{cases}$$

neutral fraction: $X_{HI} = 1 - X_{HI}$

21cmFAST simulations

matter fluctuations | neutral fraction | 21cm brightness Temperature [mK]



overview

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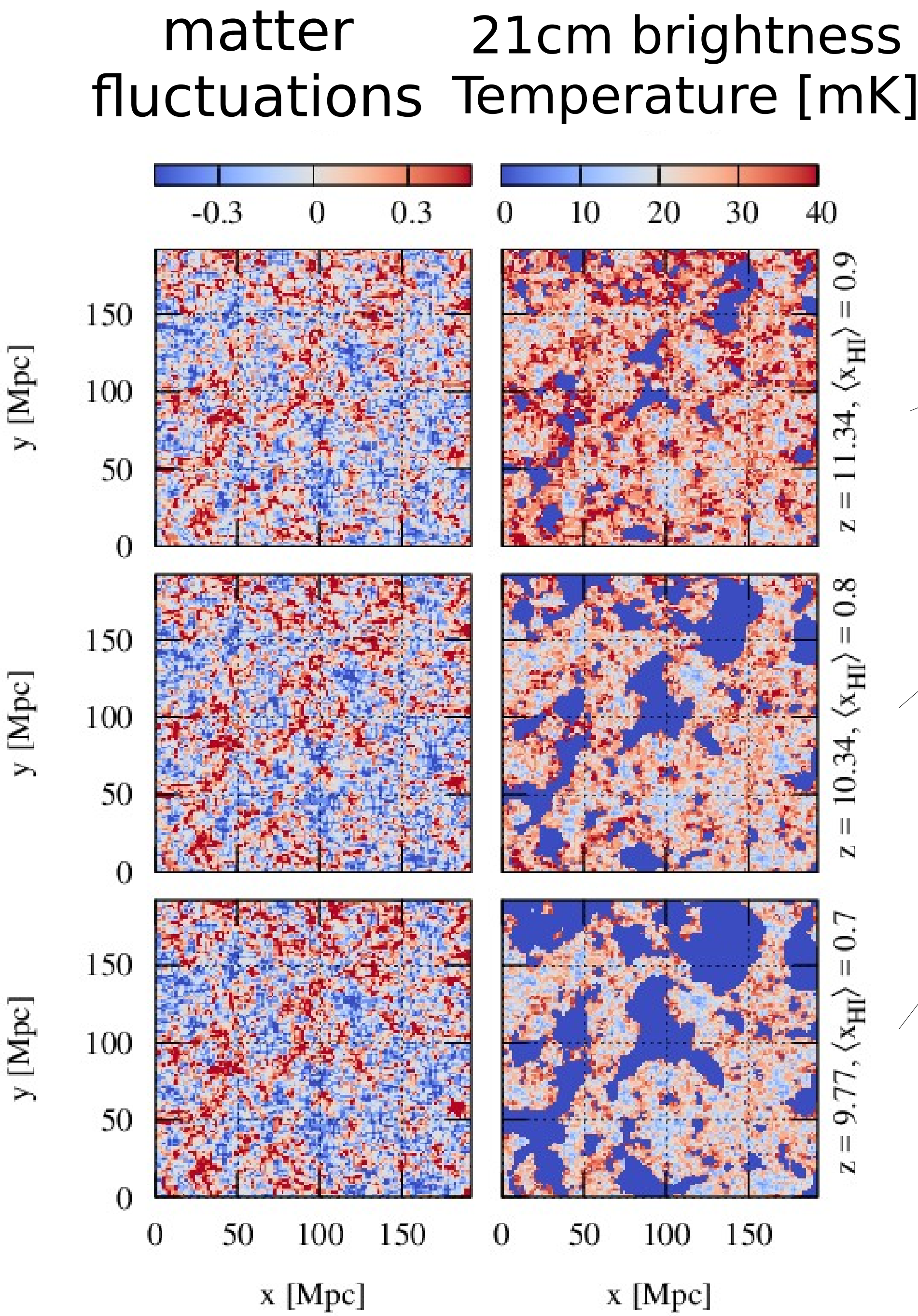
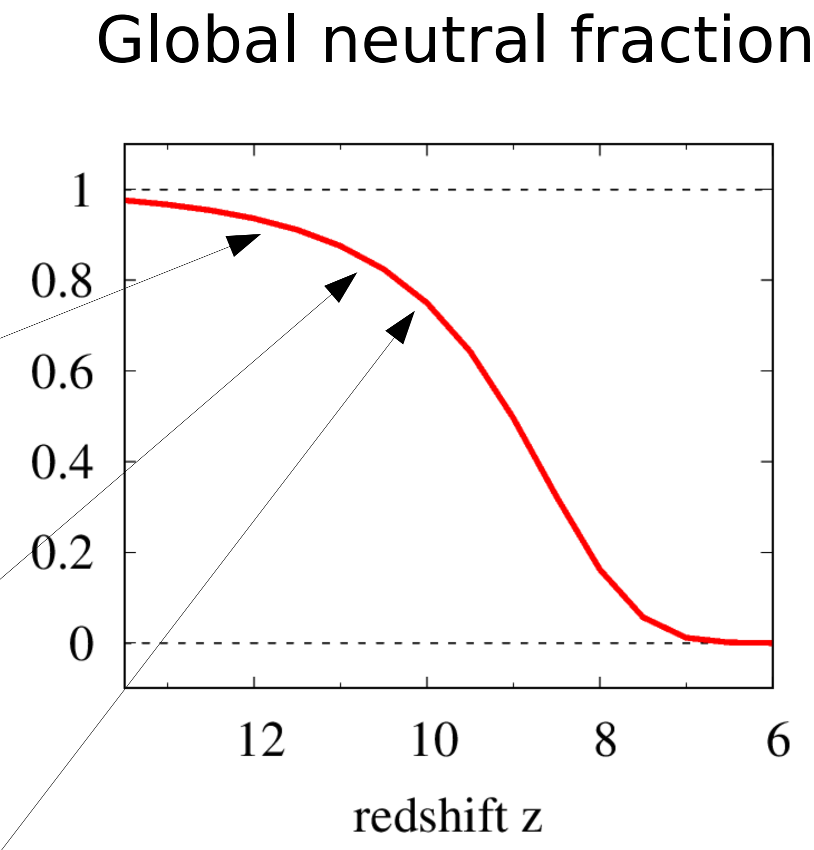
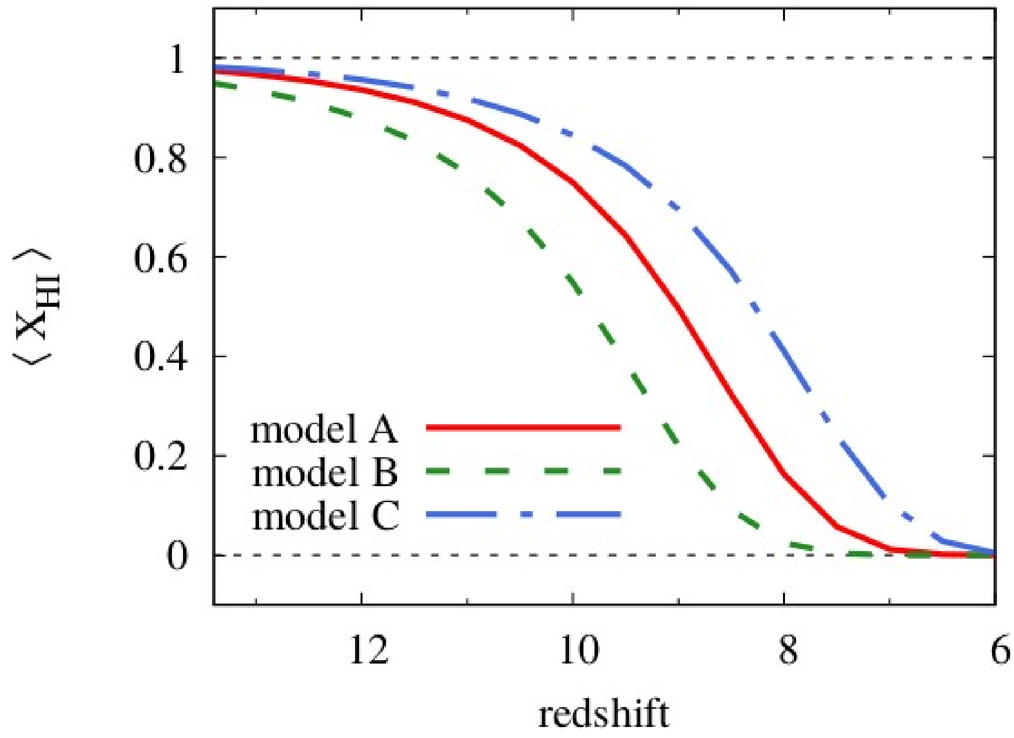
global neutral fraction $\langle X_{HI} \rangle$





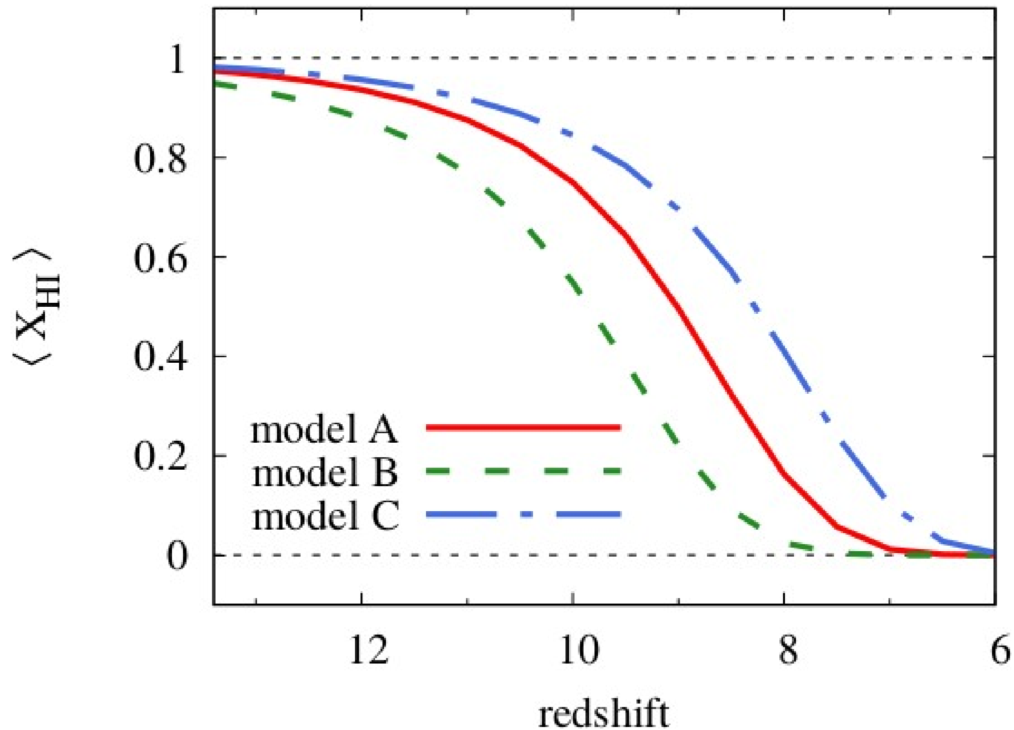
- $(768\ \mathrm{Mpc})^3$ box
- 200 realizations

dependence of global neutral fraction for different EoR model parameters

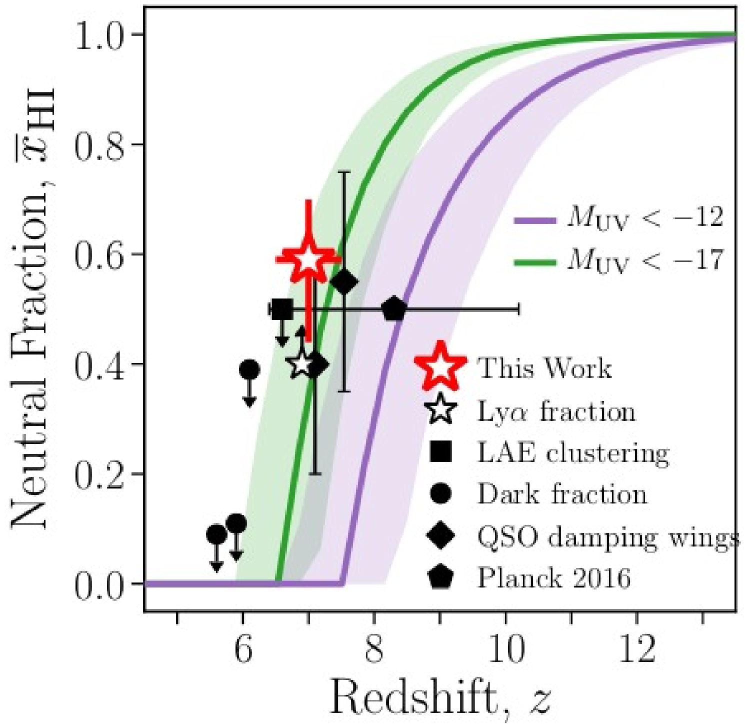


model	ζ	$T_{\min}$ [mK]
A	60	5×10^4
B	60	3×10^4
C	40	5×10^4

dependence of global neutral fraction for different EoR model parameters



observational "constraints"
Mason et al. (2018)



neutral fraction from random walks

condition for ionization

$$f_{coll} = erfc\left(\frac{\delta_{crit} - \delta_R}{[2(\sigma^2(m_{min}) - \sigma_R^2)]^{1/2}}\right) > \frac{1}{\zeta}$$

neutral fraction from random walks

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$$f_{coll} = erfc\left(\frac{\delta_{crit} - \delta_R}{[2(\sigma^2(m_{min}) - \sigma_R^2)]^{1/2}}\right) > \frac{1}{\zeta}$$

<=>

$$\delta_R > \delta_{crit} - erf^{-1}(1 - 1/\zeta)[2(\sigma^2(m_{min}) - \sigma_R^2)]^{1/2}$$

neutral fraction from random walks

condition for ionization

$$f_{coll} = erfc\left(\frac{\delta_{crit} - \delta_R}{[2(\sigma^2(m_{min}) - \sigma_R^2)]^{1/2}}\right) > \frac{1}{\zeta}$$

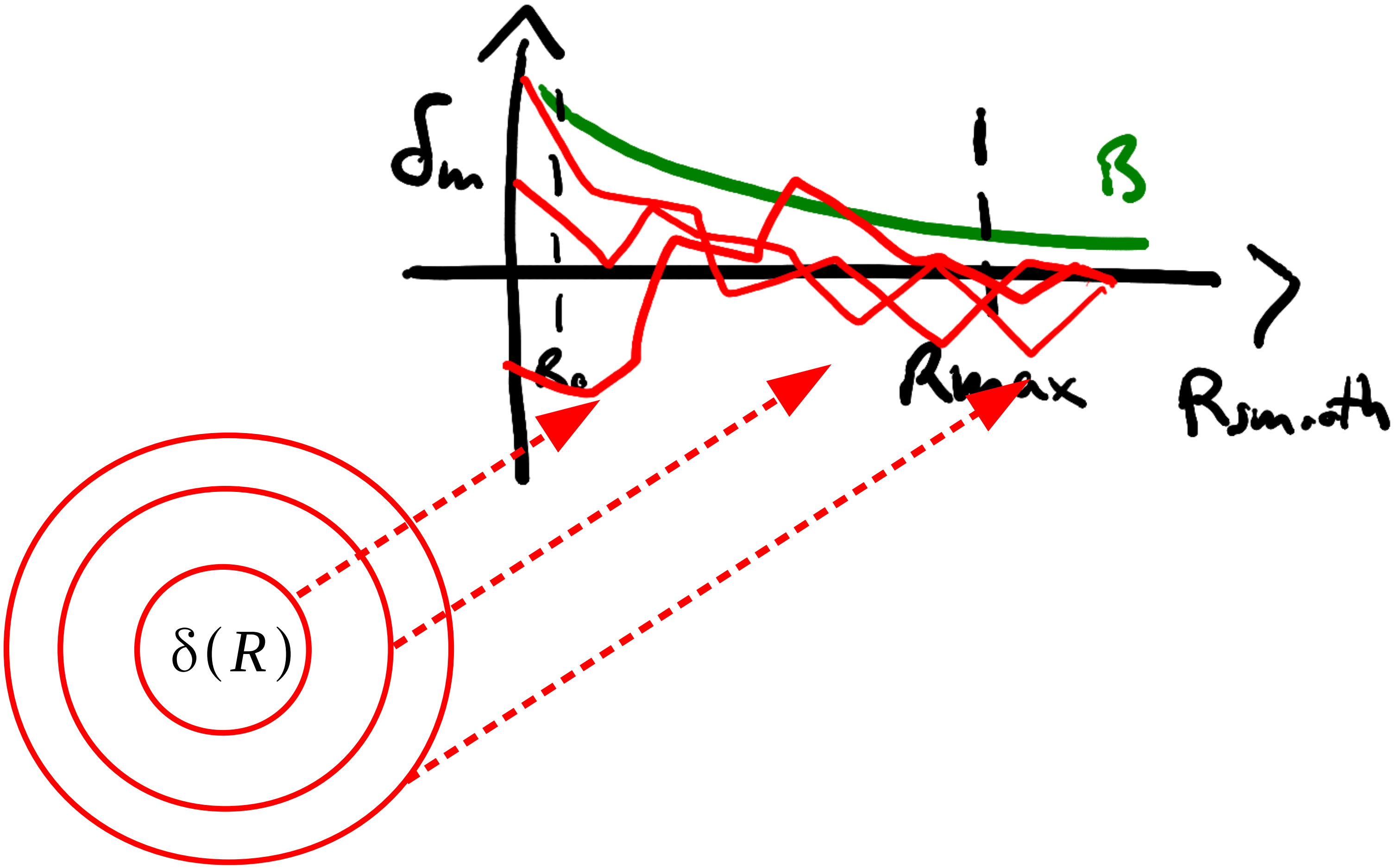
<=>

$$\delta_R > \underbrace{\delta_{crit} - erf^{-1}(1 - 1/\zeta)[2(\sigma^2(m_{min}) - \sigma_R^2)]^{1/2}}_{\text{ionization barrier}} \stackrel{\text{def}}{=} B(R)$$

neutral fraction from random walks

<u>condition for ionization</u>

$$\delta_R > \delta_{crit} - erf^{-1}(1-1/\zeta)\left[2\left(\sigma^2(m_{min}) - \sigma_R^2\right)\right]^{1/2} \stackrel{\text{def}}{=} B$$



simulating random walks

- generating random walks using predictions for variance and correlation between steps

  $$\delta_m(R_{i+1}) = \delta_m(R_i) + \Delta_i$$

  high redshifts and large smoothing scales
  => assume linear and Gaussian matter field

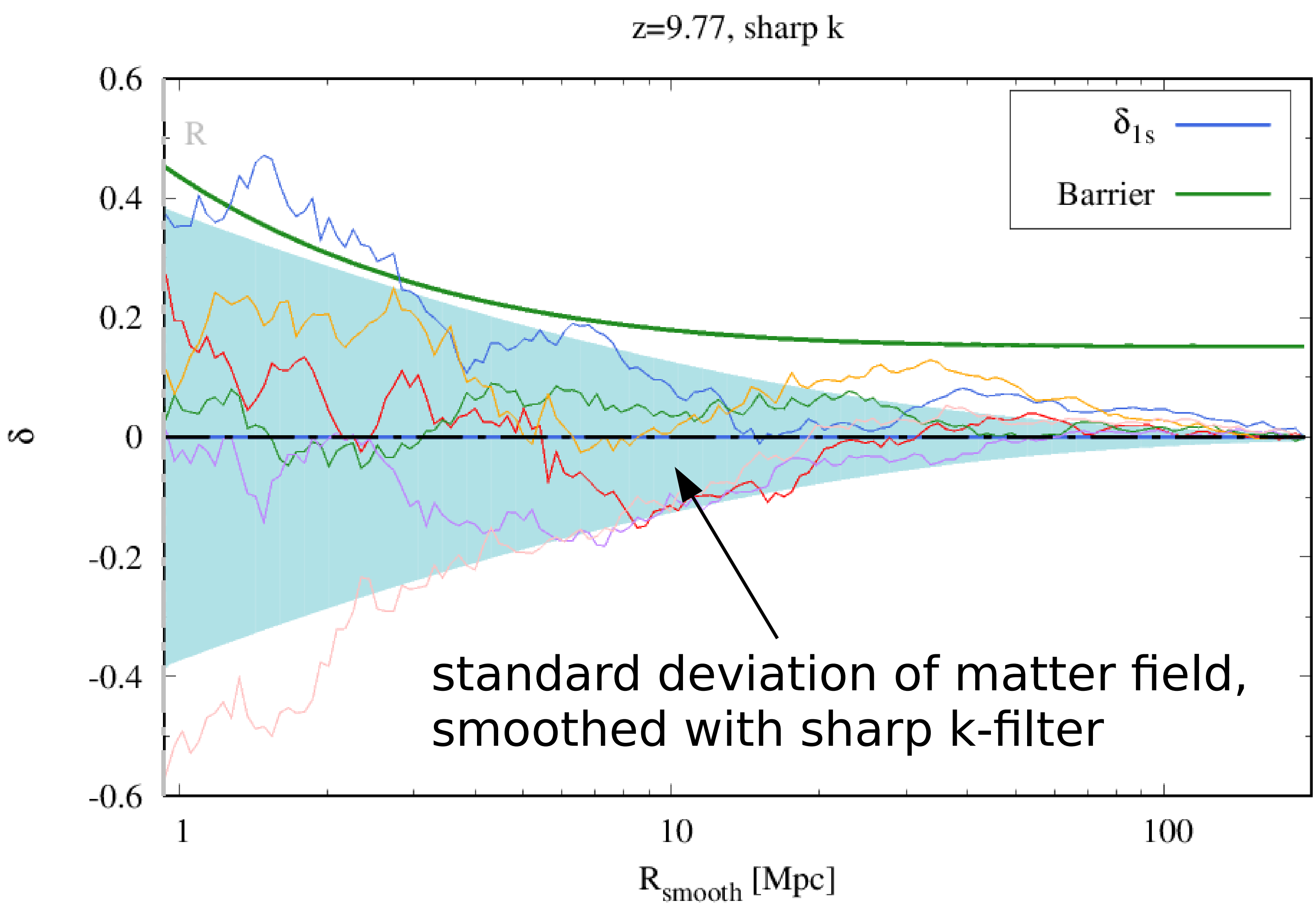
- uncorrelated steps for Gaussian matter field and skarp-k smoothing window W in Fourier space

  $$\langle \Delta_i \Delta_{i+1} \rangle = 0$$

- variance from linear power spectrum

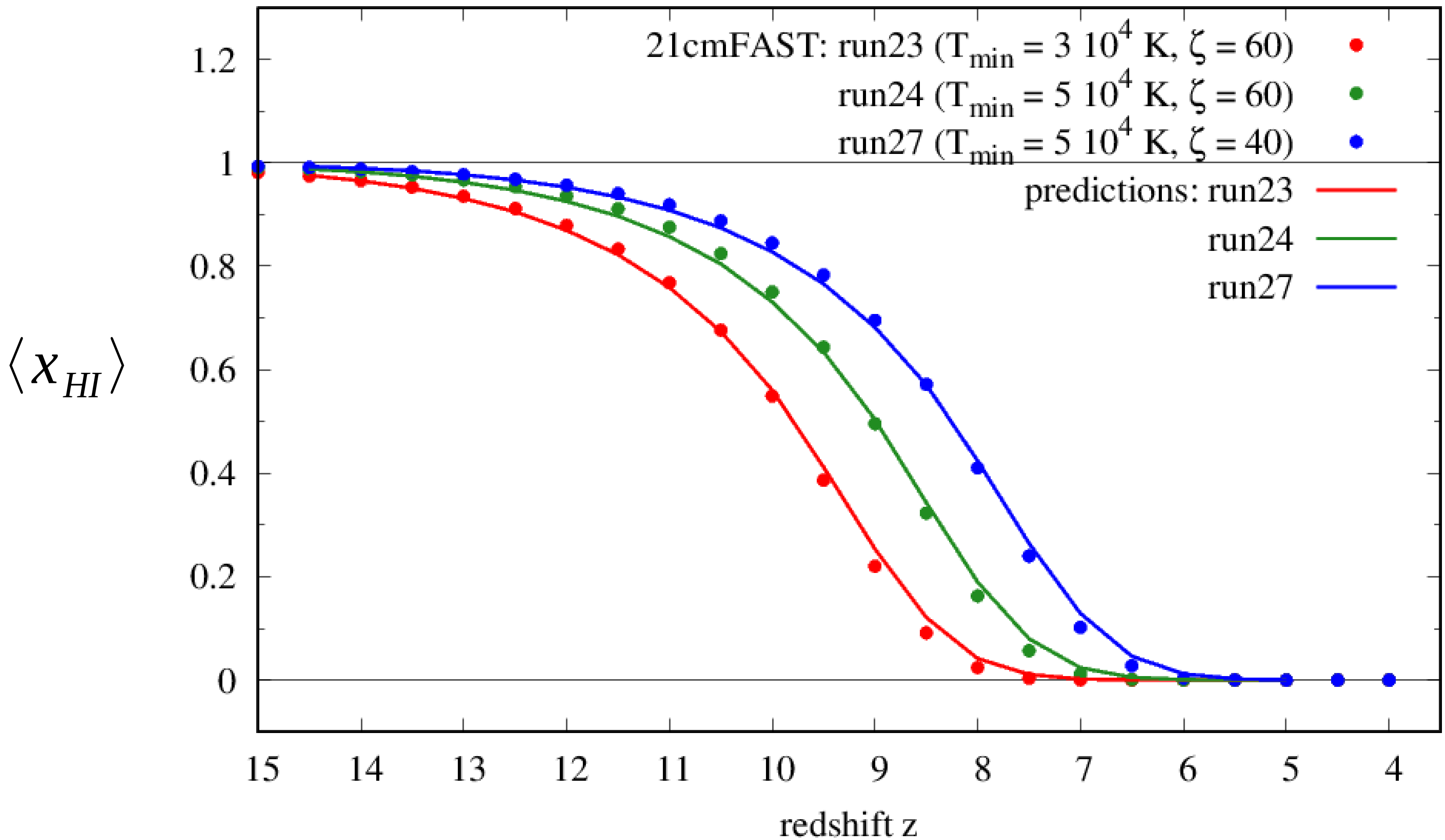
  $$\langle \delta_m^2(R_i) \rangle = \int_0^\infty \frac{dk}{k} \frac{k^3 P_{lin}(k)}{2\pi^2} W^2(k R_i)$$

simulating random walks



neutral fraction =
fraction of walks, which never cross the barrier

simulating random walks



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2- and 3-point correlations

fluctuations:

$$\delta_m = \frac{\rho_m - \langle \rho_m \rangle}{\langle \rho_m \rangle} \qquad \delta_{\delta T} = \frac{\delta T - \langle \delta T \rangle}{\langle \delta T \rangle}$$

2- and 3-point correlations

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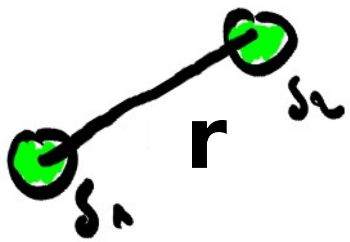
$$\delta_m = \frac{\rho_m - \langle \rho_m \rangle}{\langle \rho_m \rangle} \qquad \delta_{\delta T} = \frac{\delta T - \langle \delta T \rangle}{\langle \delta T \rangle}$$

2-point correlation (2pc):

$$\xi^{(12)} \stackrel{\text{def}}{=} \langle \delta_1 \delta_2 \rangle (r)$$



- spherically symmetric →not sensitive to shape of fluctuations

2- and 3-point correlations

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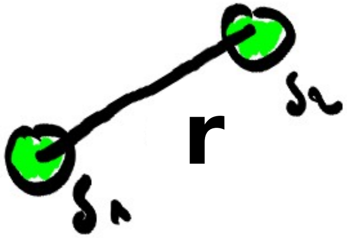

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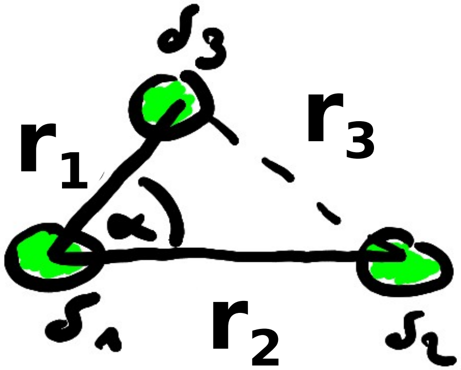
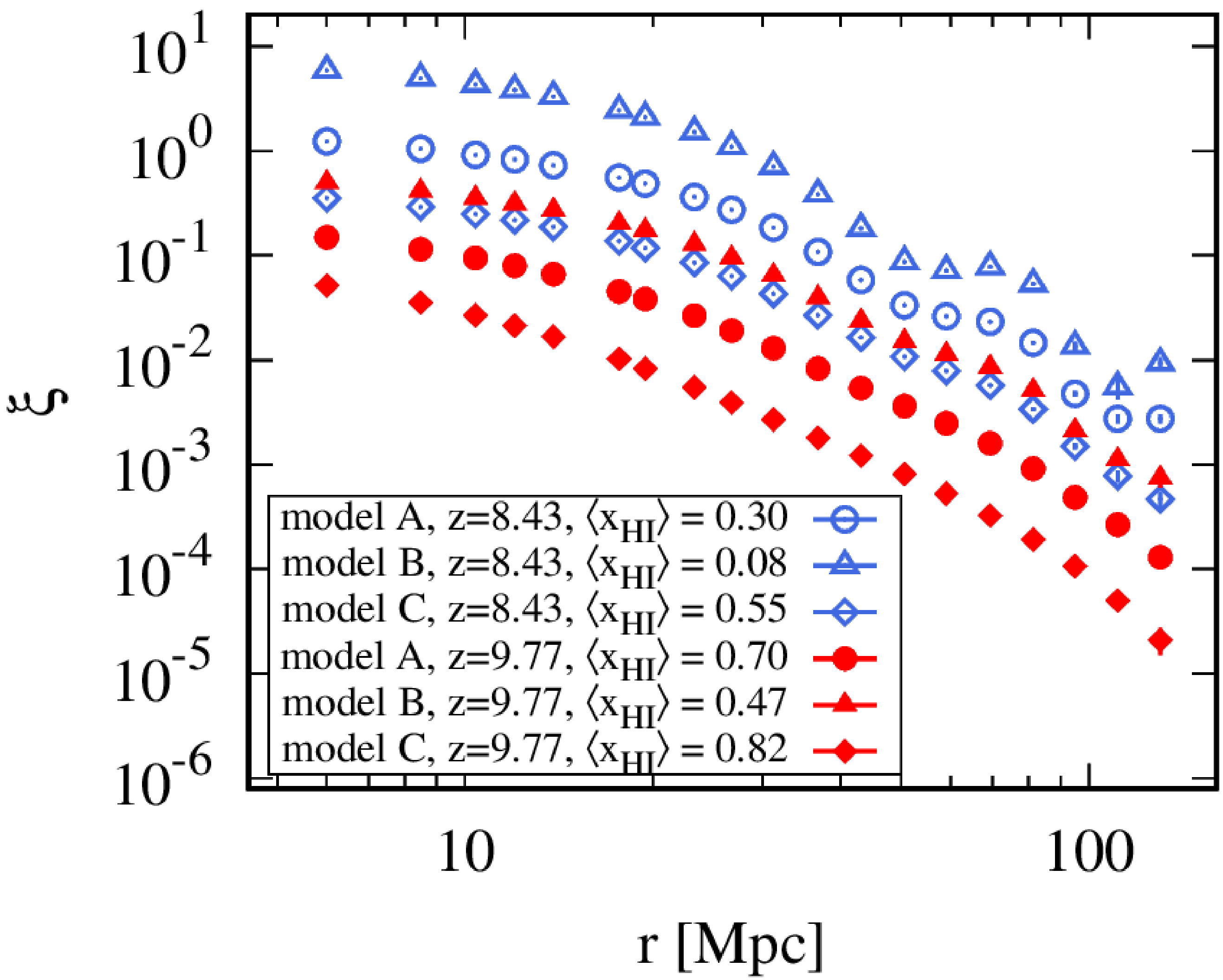
3-point correlation (3pc):



$$\zeta^{(123)} \stackrel{\text{def}}{=} \langle \delta_1 \delta_2 \delta_3 \rangle (r_{1,} r_{2,} r_3)$$

- provides additional shape info

21cm 2pc measurements



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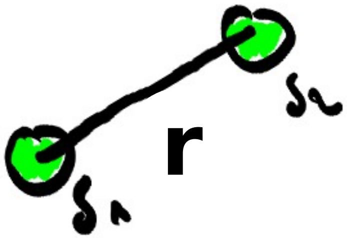
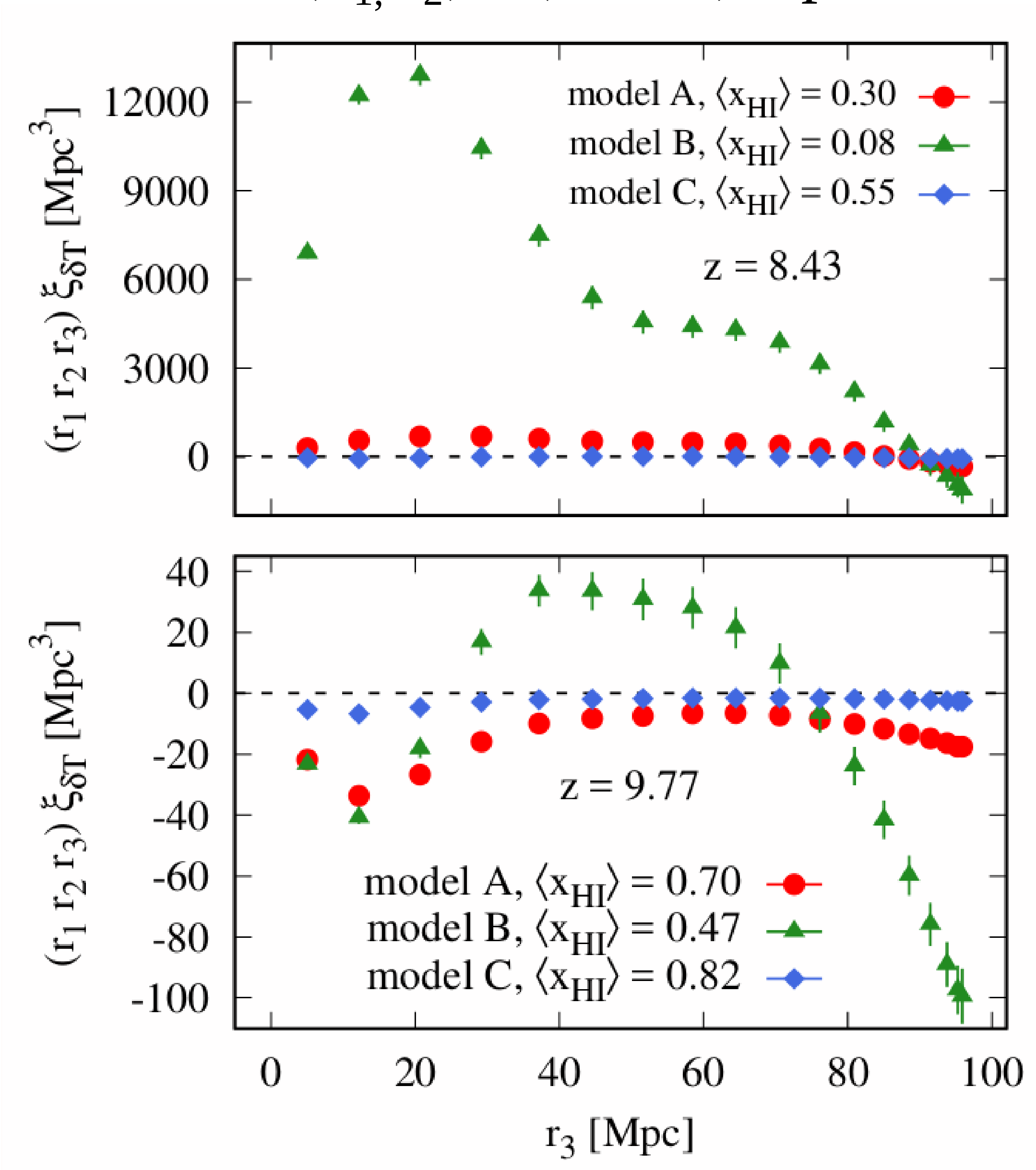


$z = 8.43$

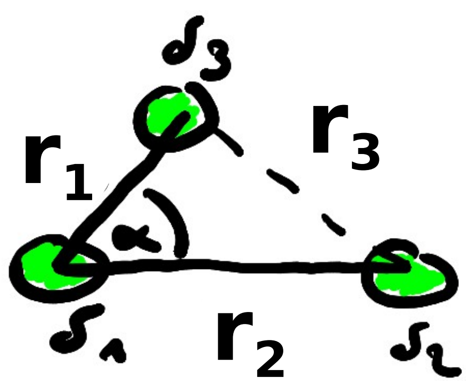
$z = 9.77$

21cm 3pc measurements

$$(r_1, r_2) = (48, 48)\,Mpc$$



$$\zeta^{(123)} \stackrel{\text{def}}{=} \langle \delta_1 \delta_2 \delta_3 \rangle (r_1, r_2, \alpha)$$



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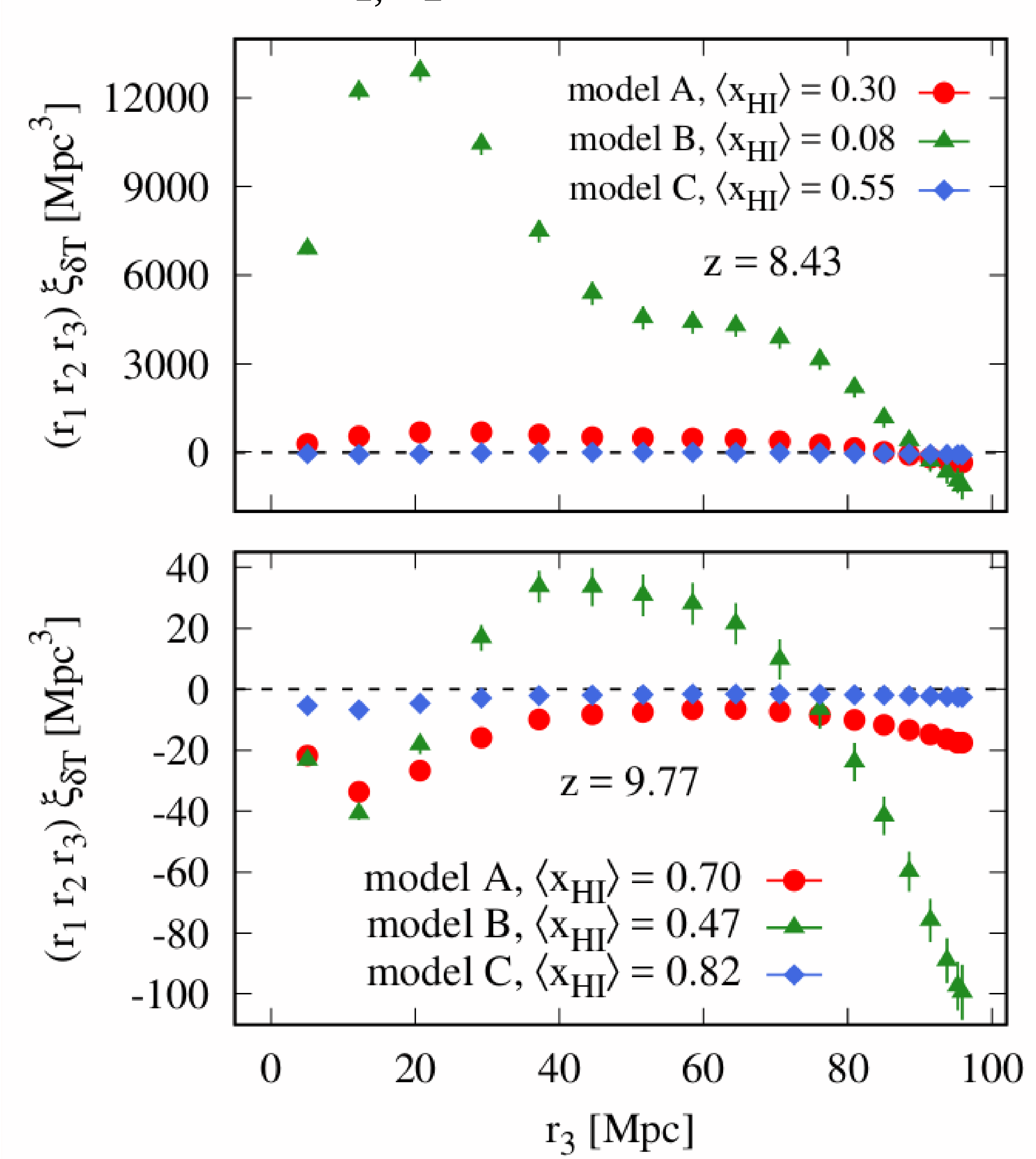

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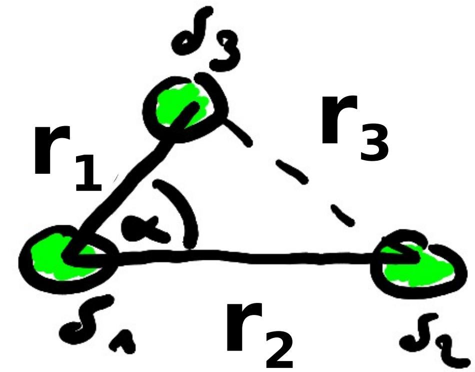


recent literature

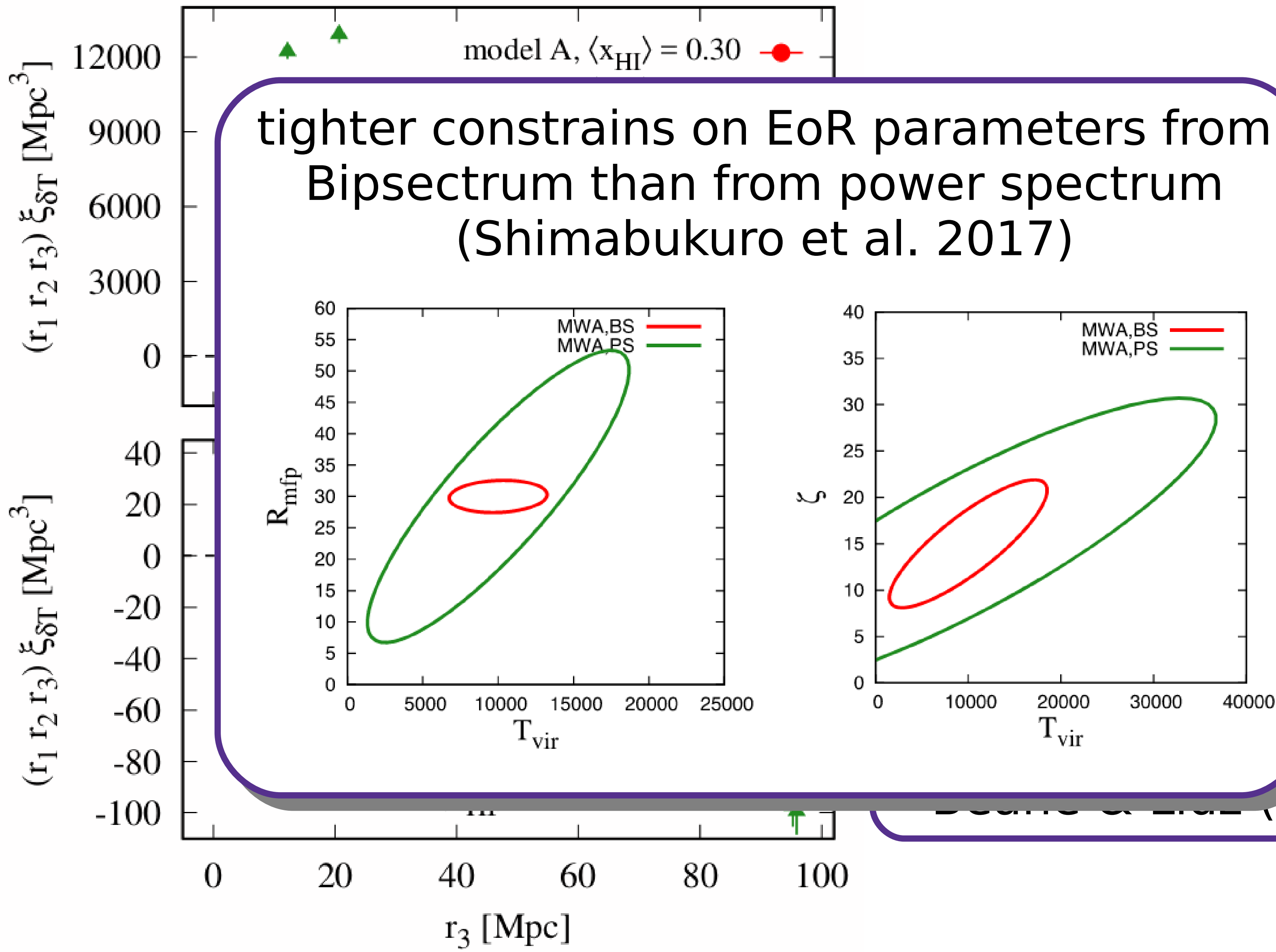
- Shimabukuro et al. (2016)
- Majumdar et al. (2017)
- Shimabukuro et al. (2017)
- Watkinson et al. (2018)
- Beane & Lidz (2018)

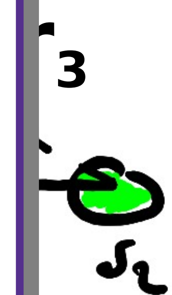
21cm 3pc measurements

$$(r_1, r_2) = (48{,}48)\ Mpc$$

$$\zeta^{(123)} \stackrel{\text{def}}{=} \langle \delta_1 \delta_2 \delta_3 \rangle (r_1, r_2, \alpha)$$

tighter constrains on EoR parameters from Bipsectrum than from power spectrum (Shimabukuro et al. 2017)





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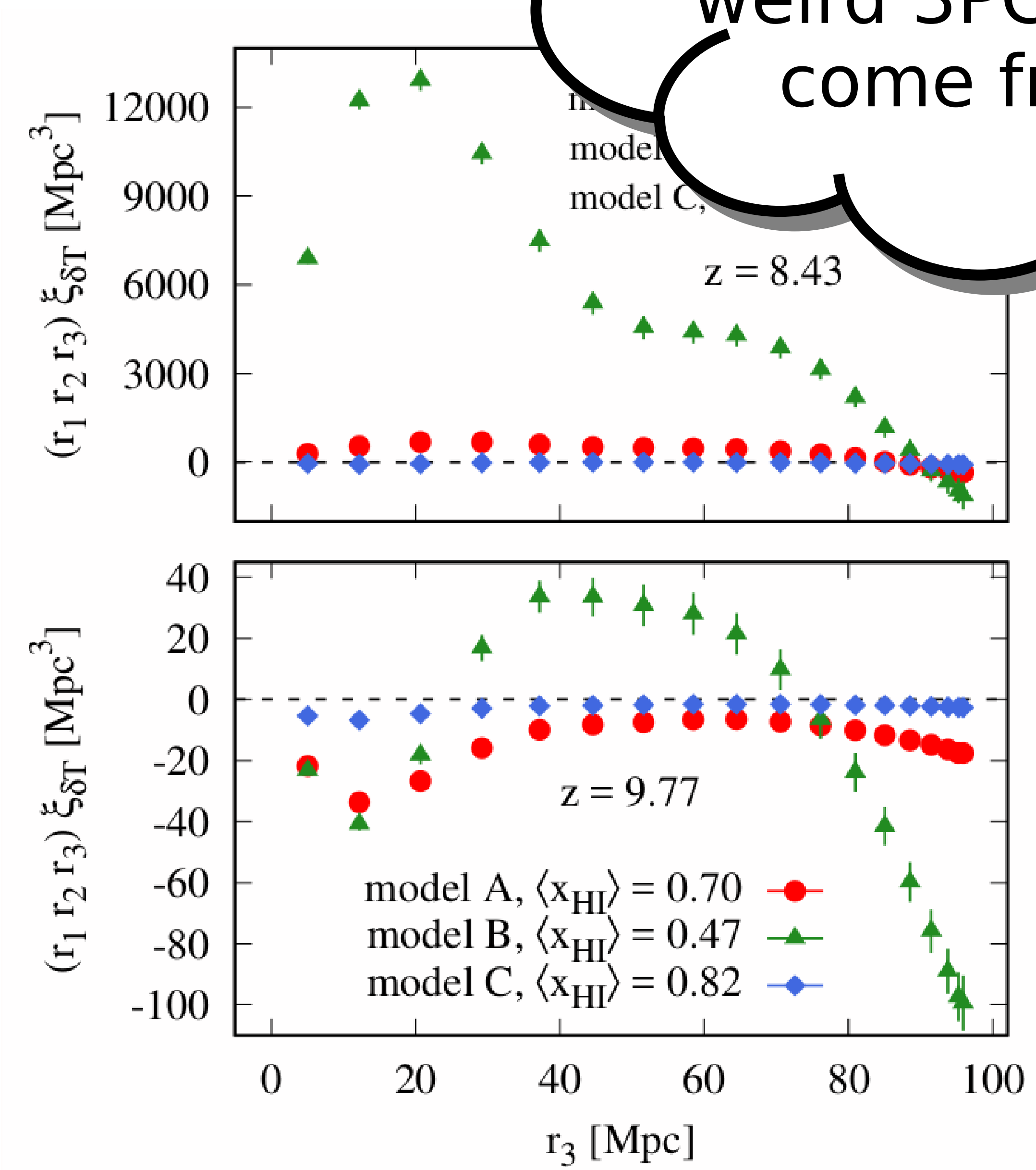
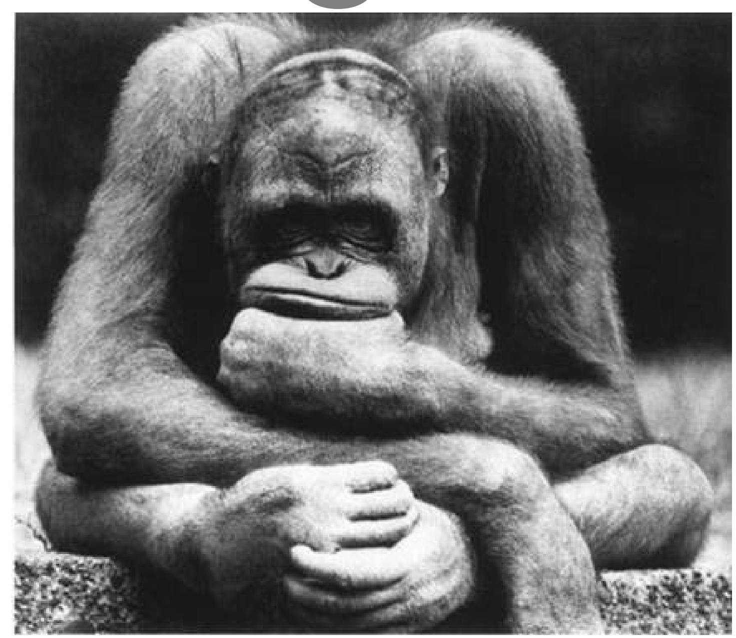
al. (2016)
(2017)
al. (2017)
(2018)
(2018)

21cm 3r urements

Where does that weird 3PC shape come from?





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local quadratic bias model

Assumption: 21cm brightness temperature is **deterministic** function of underlying matter density

$$\delta_{\delta T} = F(\delta_m) \approx \sum_{n=0}^{N} b_n \frac{\delta_m^n}{n!}$$

(Taylor expansion of F around $\delta_m \approx 0$)

local quadratic bias model

Assumption: 21cm brightness temperature is **deterministic** function of underlying matter density

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(Taylor expansion of F around $\delta_m \approx 0$)

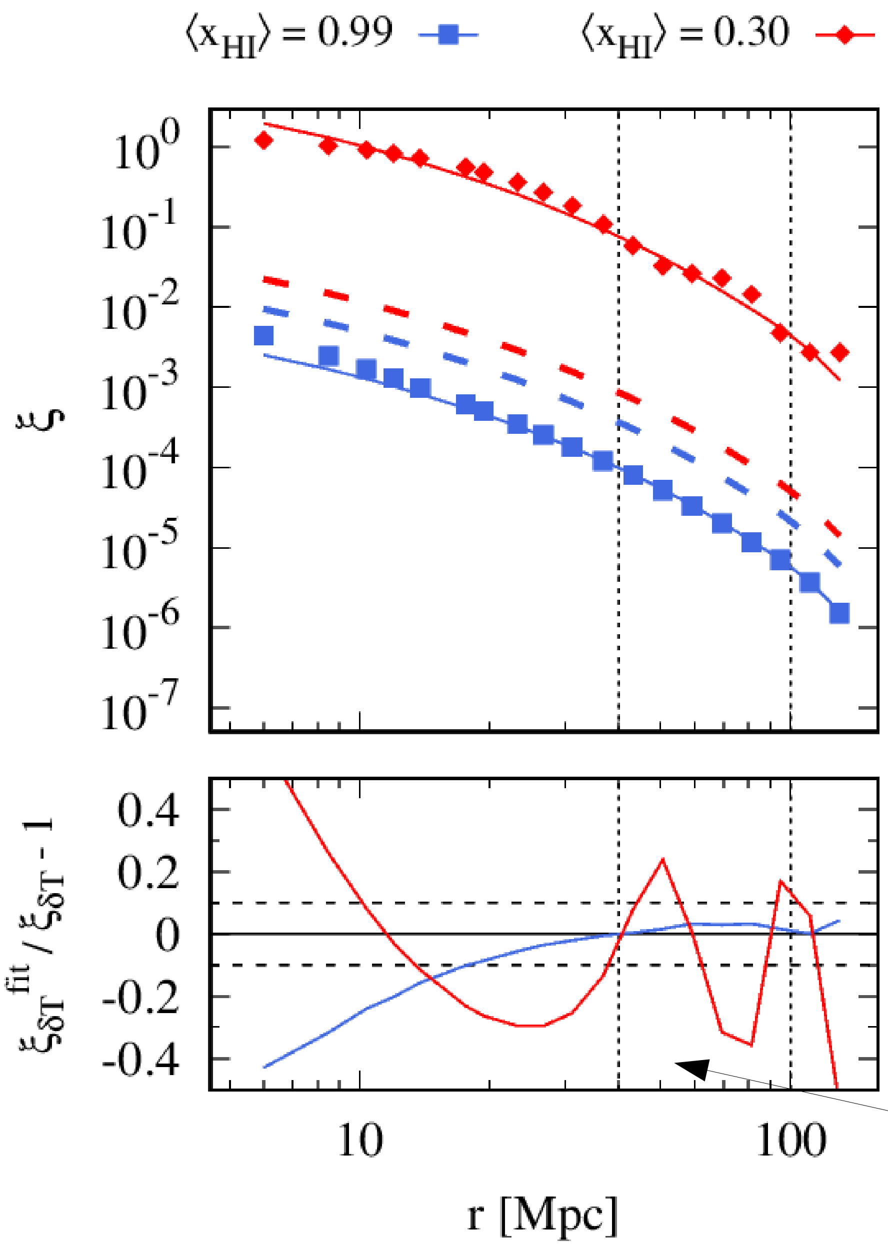
Leading-order approximations for correlation functions

2pc $\xi_{\delta T} \approx b_1^2 \xi_m$

3pc $\zeta_{\delta T} \approx b_1^3 \zeta_m + b_1^2 b_2 \overbrace{(\xi_{12}\xi_{13} + 2\, perm.)}^{\zeta_m^H}$

(Fry & Gaztanaga 93)

2pc bias modeling



mean measurements
over 200 realizations

symbols: 21cm 2pc ($\xi_{\delta T}$)

dashed lines: matter 2pc (ξ_m)

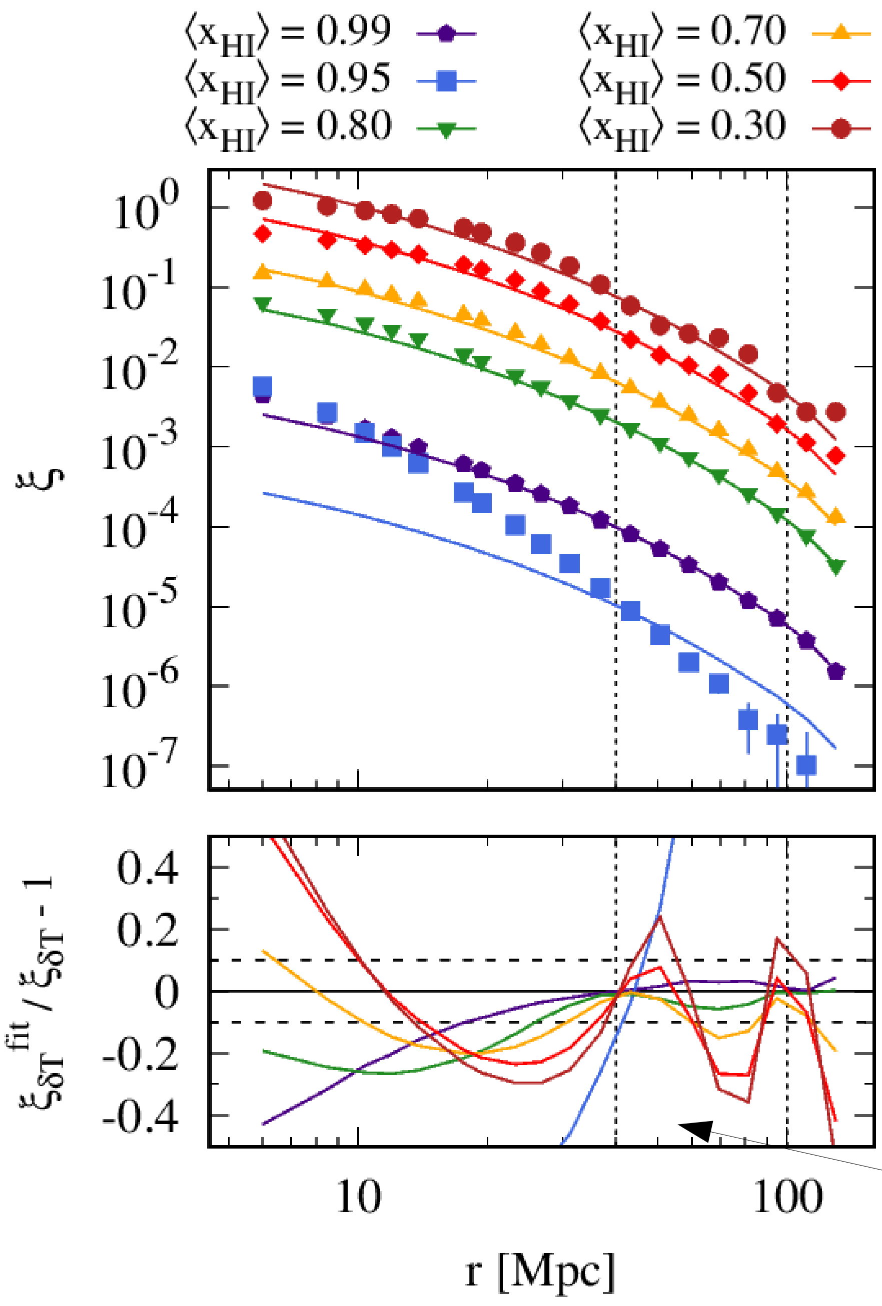
solid lines: fits to bias model

bias model prediction

$$\xi_{\delta T} = b_1^2 \xi_m$$

fitting range: 40 < r < 90 Mpc

2pc bias modeling



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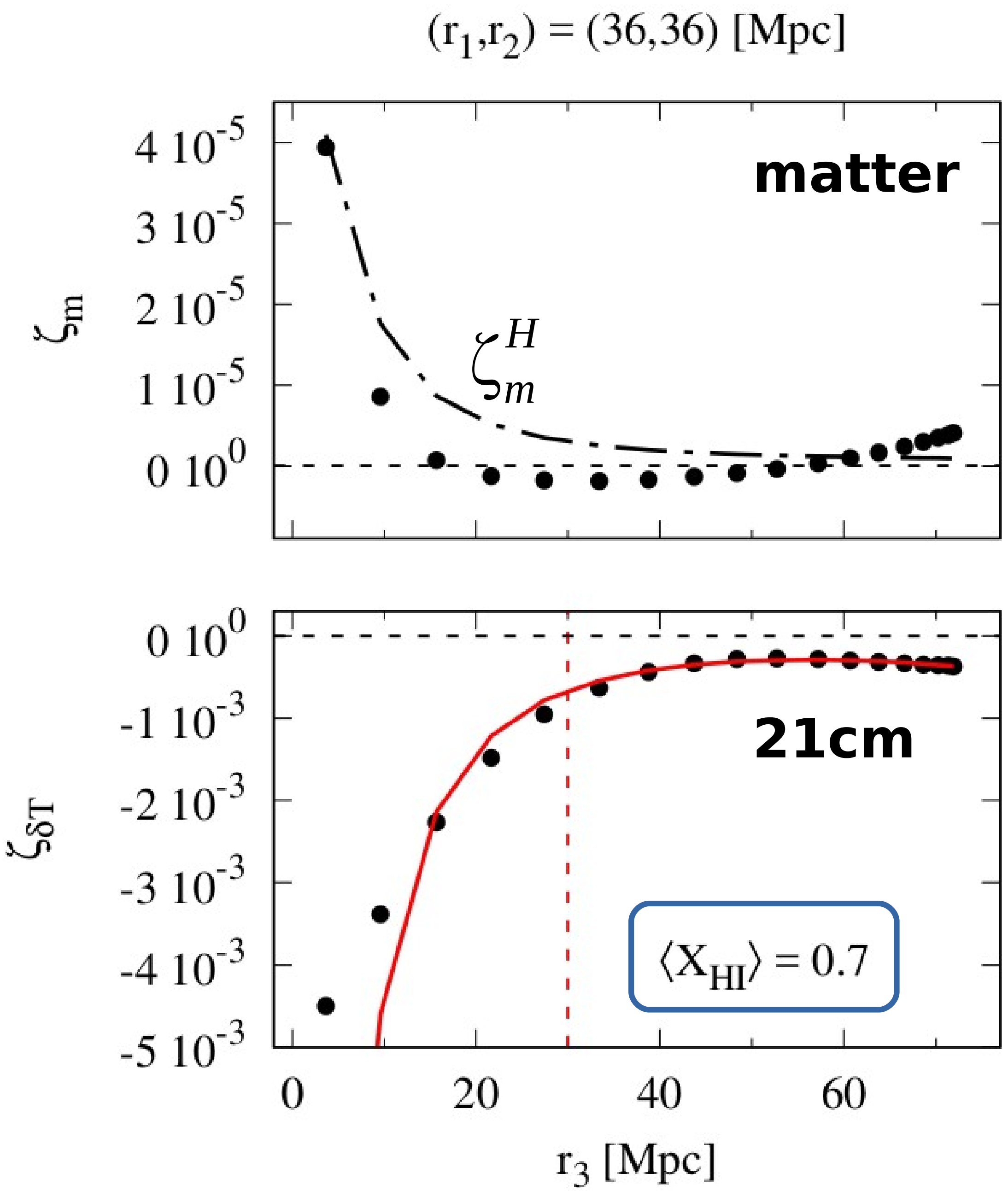
3pc bias modeling



mean measurements over 200 realizations

dots: 3pc measurements

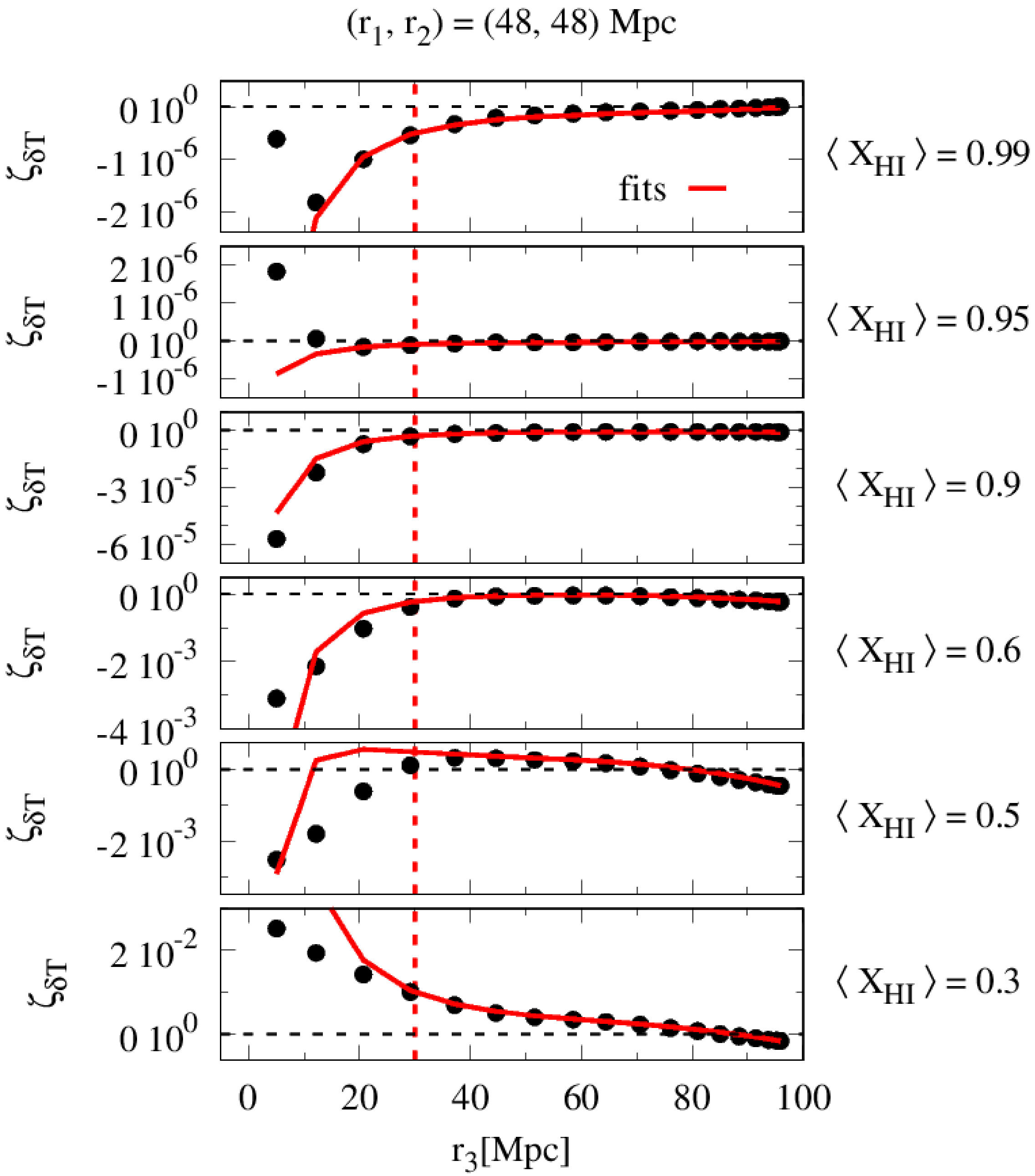
solid line: fit to bias model

bias model prediction

$$\zeta_{\delta T} \approx b_1^3 \zeta_m + b_1^2 b_2 \zeta_m^H$$

(fitting range: 30<r<90 Mpc)

3pc bias modeling



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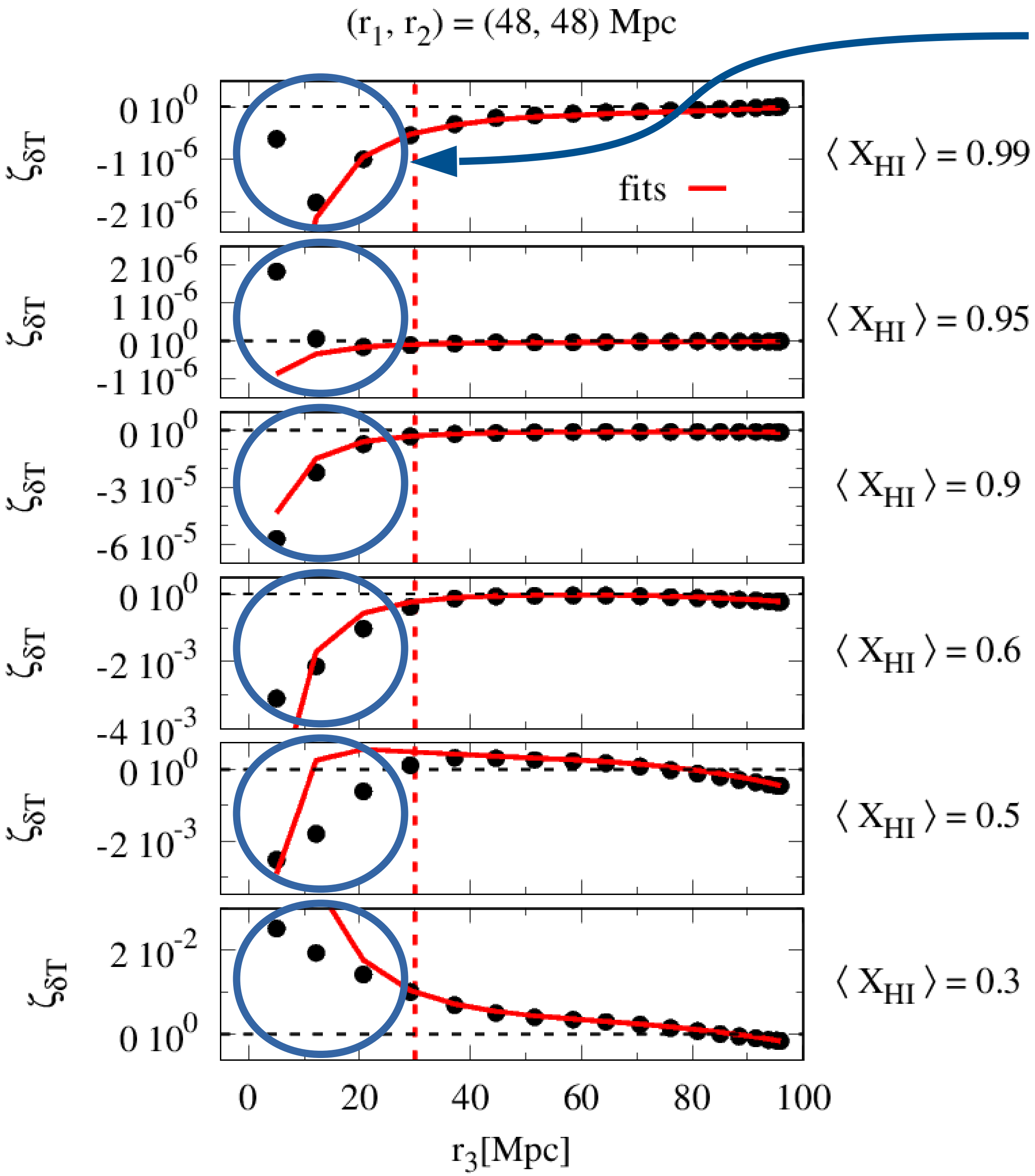
(fitting range: 30<r<90 Mpc)

3pc bias modeling



model fails at r3 < 20 Mpc

mean measurements over 200 realizations

dots: 3pc measurements

solid line: fit to bias model

bias model prediction

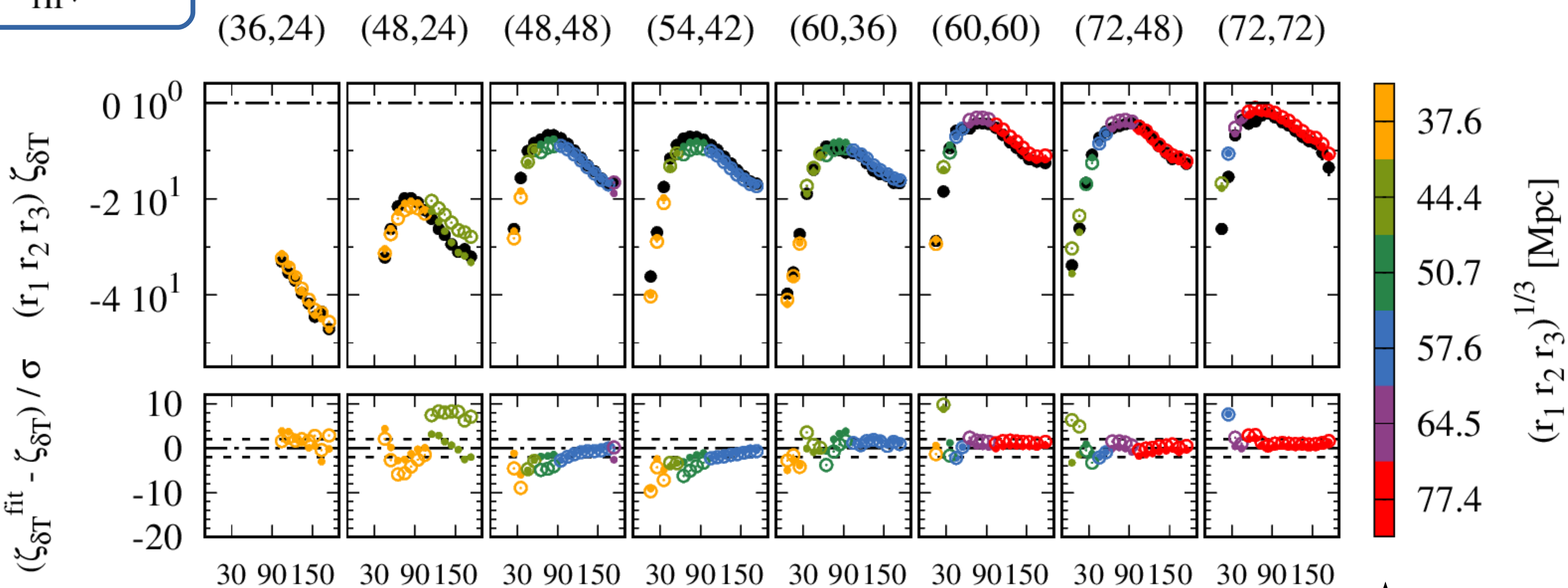
$$\zeta_{\delta T} \approx b_1^3 \zeta_m + b_1^2 b_2 \zeta_m^H$$

(fitting range: 30<r<90 Mpc)

21cm 3pc fits

$\langle X_{HI} \rangle = 0.7$

fixed triangle legs (r1, r2)



triangle opening angle

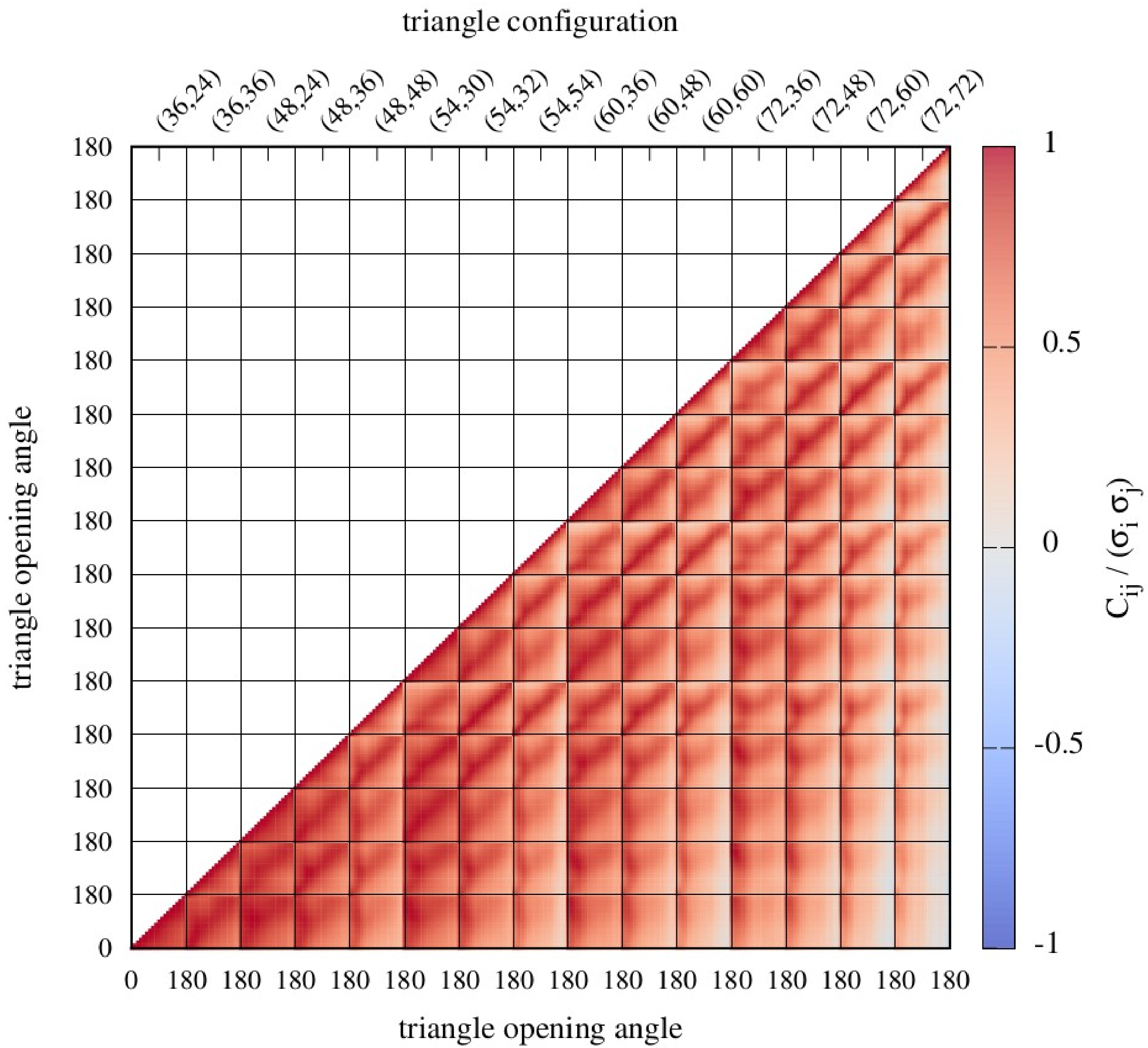
- **black dots:** 3pc measurements
- **colored dots:** fits to bias model prediction in triangle scale bins

triangle scale

$(r_1 r_2 r_3)^{1/3}$

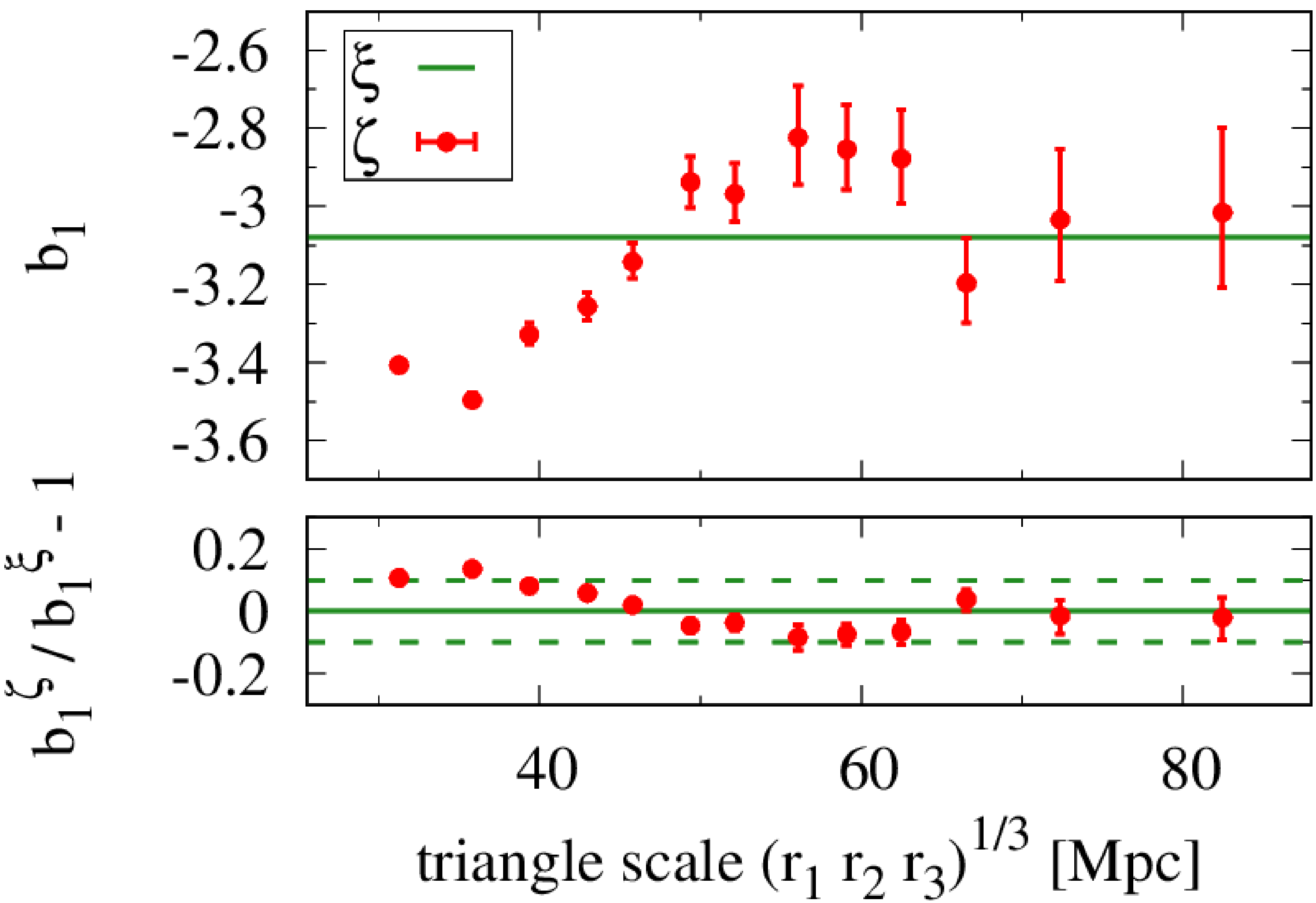
21cm 3pc covariance

15 configurations x
18 opening angles
= 270 triangles



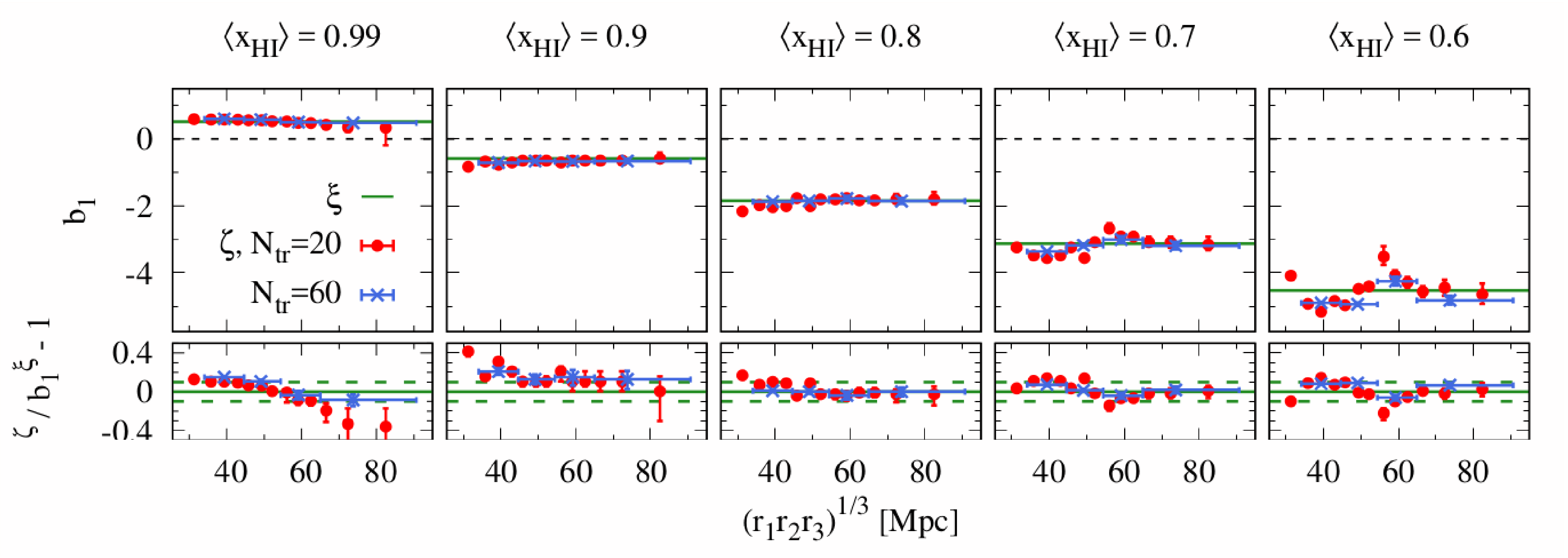
linear bias measurements

$\langle x_{HI} \rangle = 0.7$



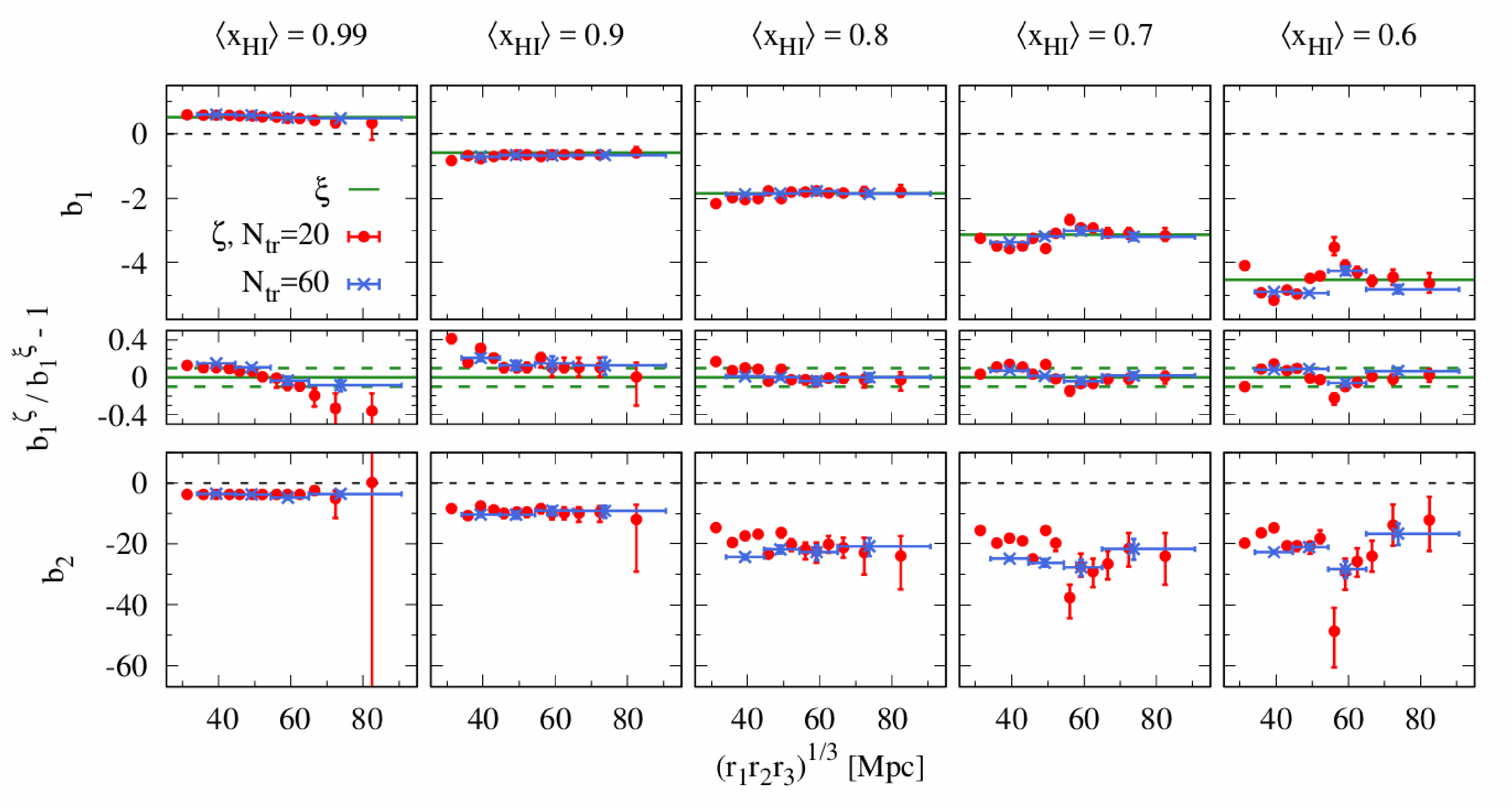
~10% agreement between linear bias from 2pc and 3pc

linear bias measurements

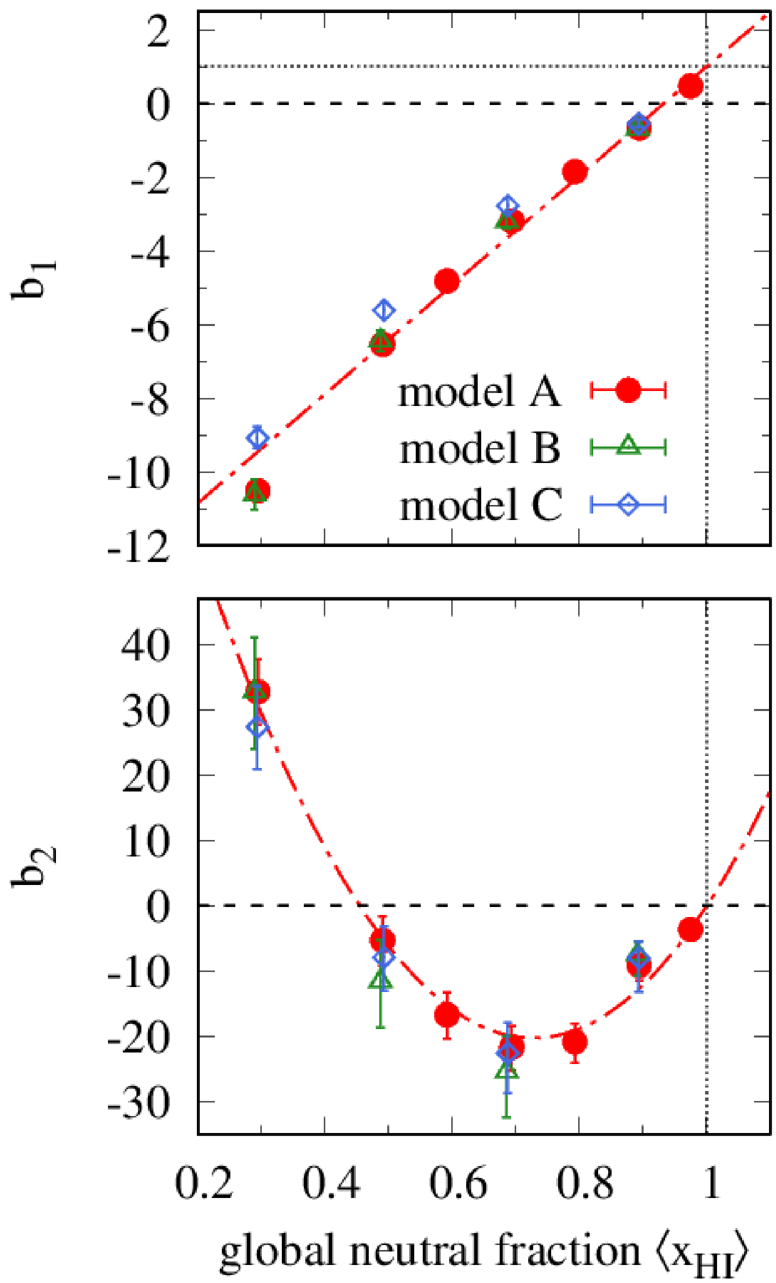


~10% agreement between linear bias from 2pc and 3pc

linear bias measurements



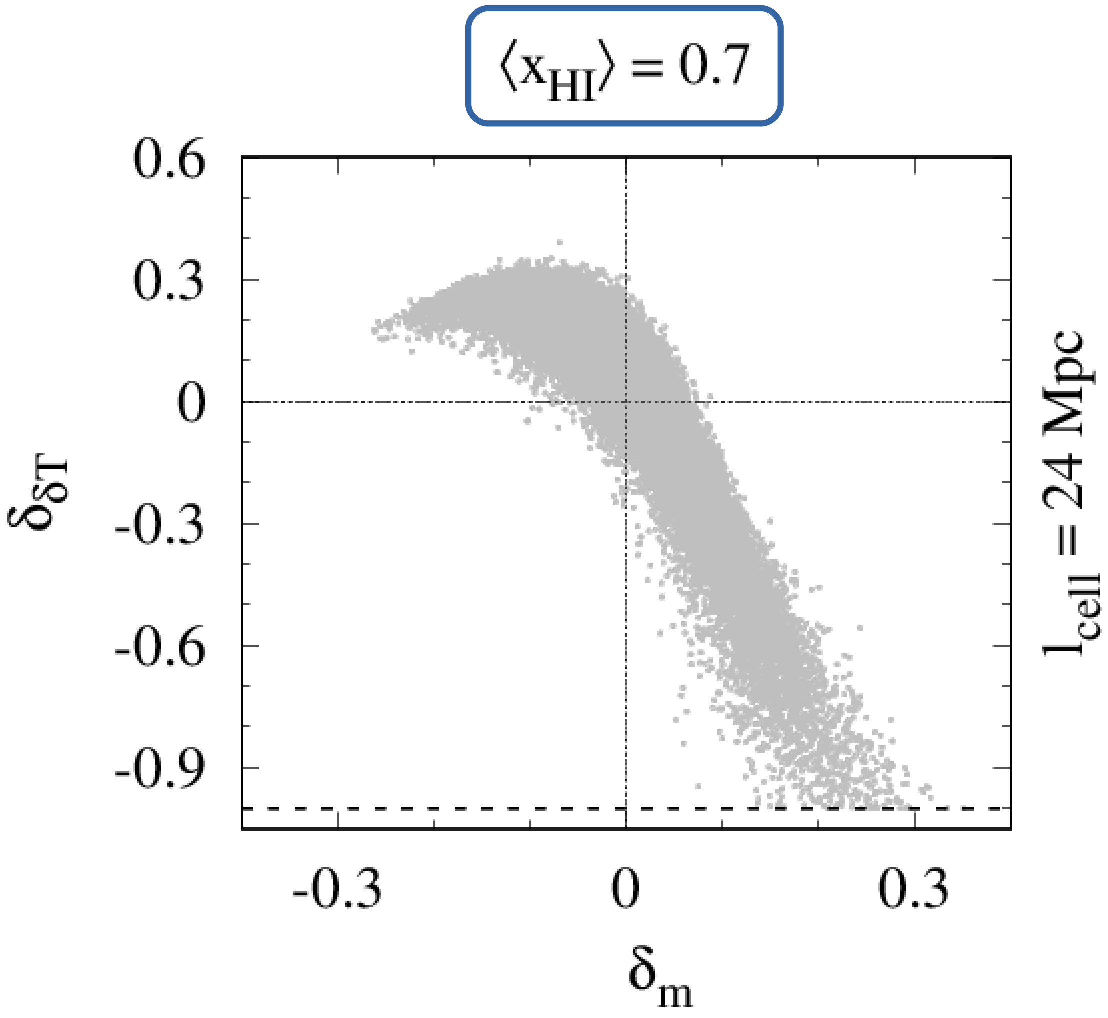
fun fact



- linear and quadratic bias follow linear and quadratic relation to global neutral fraction

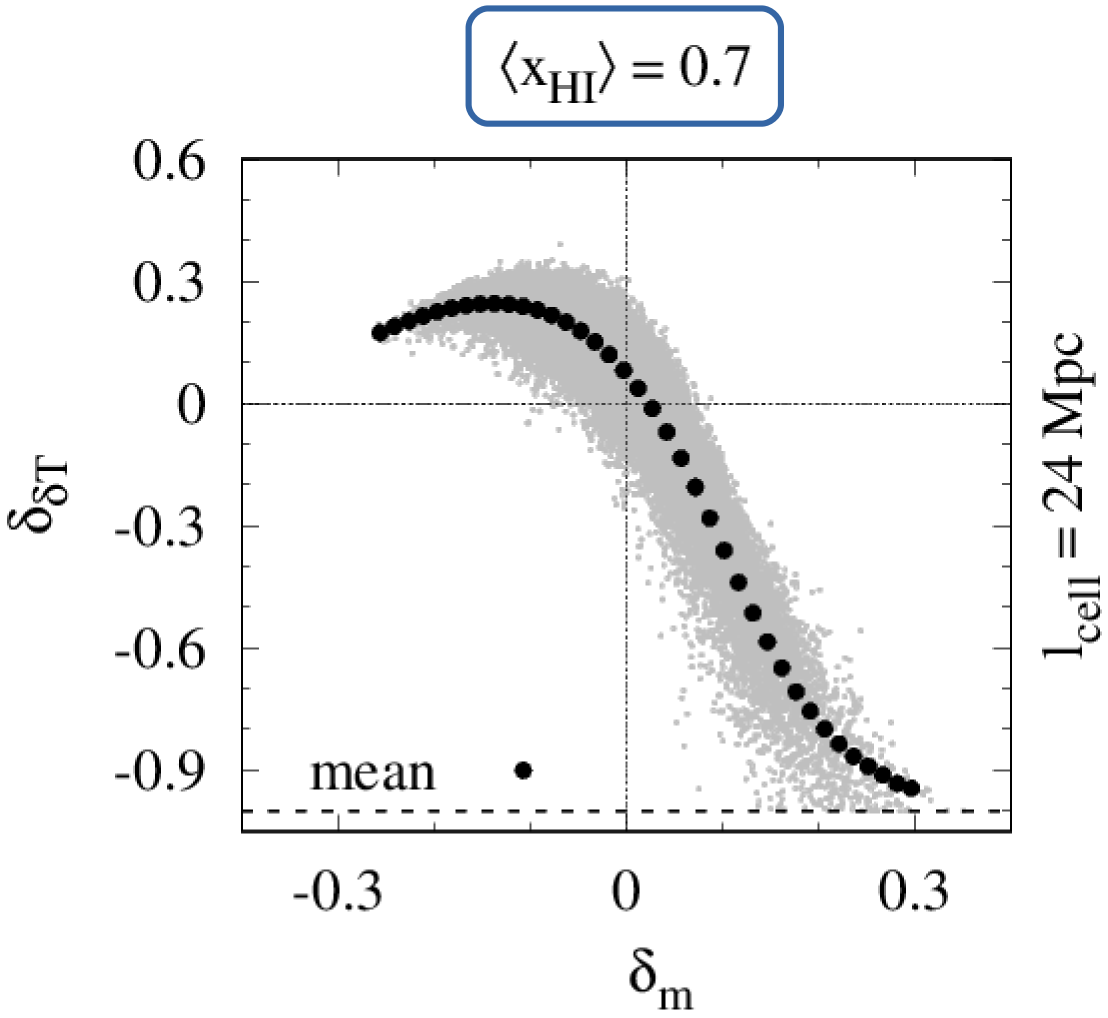
- not clear why..

bias model validation



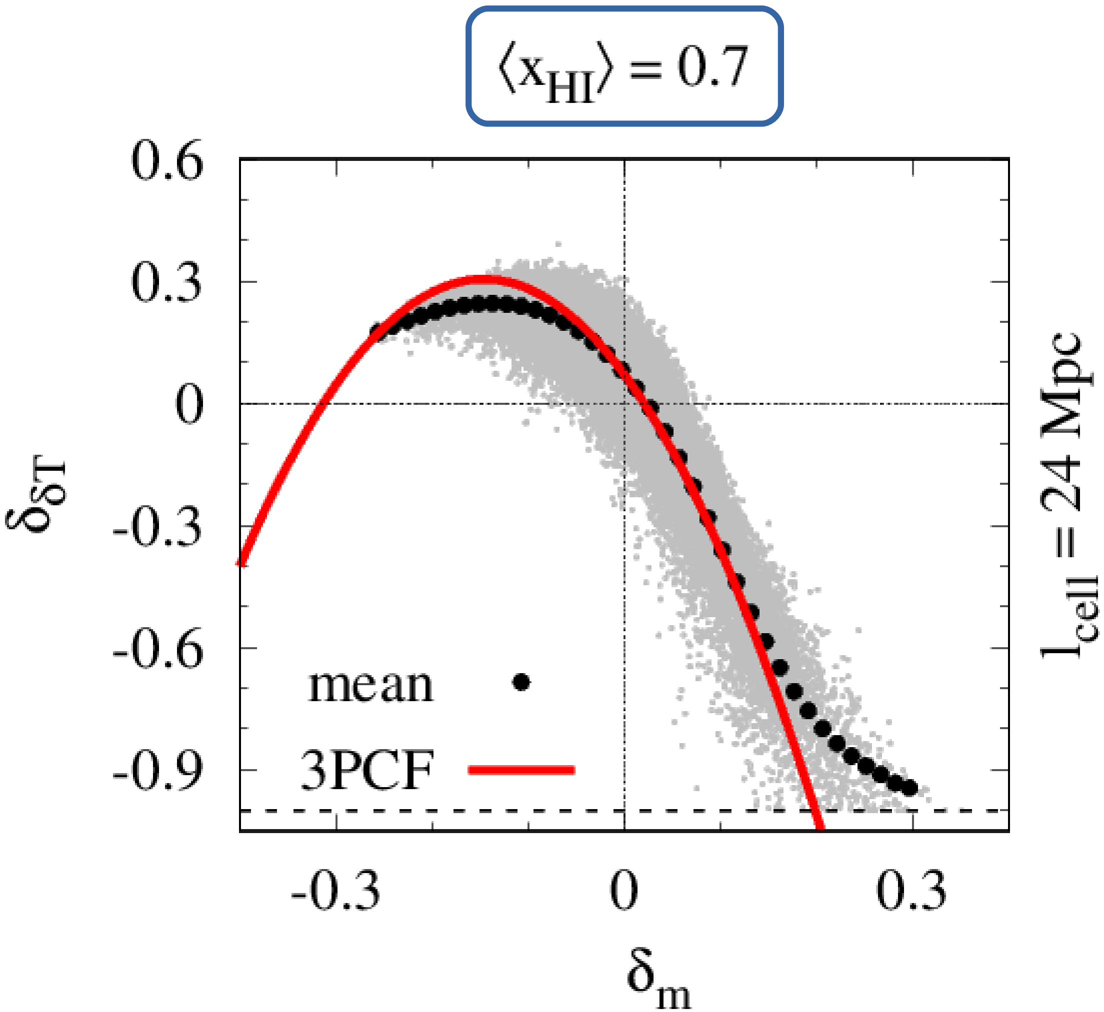
- **grey dots:** fluctuations in 24 Mpc cubical grid cells in one realization

bias model validation



- **grey dots:** fluctuations in 24 Mpc cubical grid cells in one realization
- **black dots:** mean $\delta_{\delta T}$ in bins of δ_m

bias model validation



- **grey dots:** fluctuations in 24 Mpc cubical grid cells in one realization
- **black dots:** mean $\delta_{\delta T}$ in bins of δ_m
- **red line:** quadratic bias model with b1 and b2 from 3pc measurements

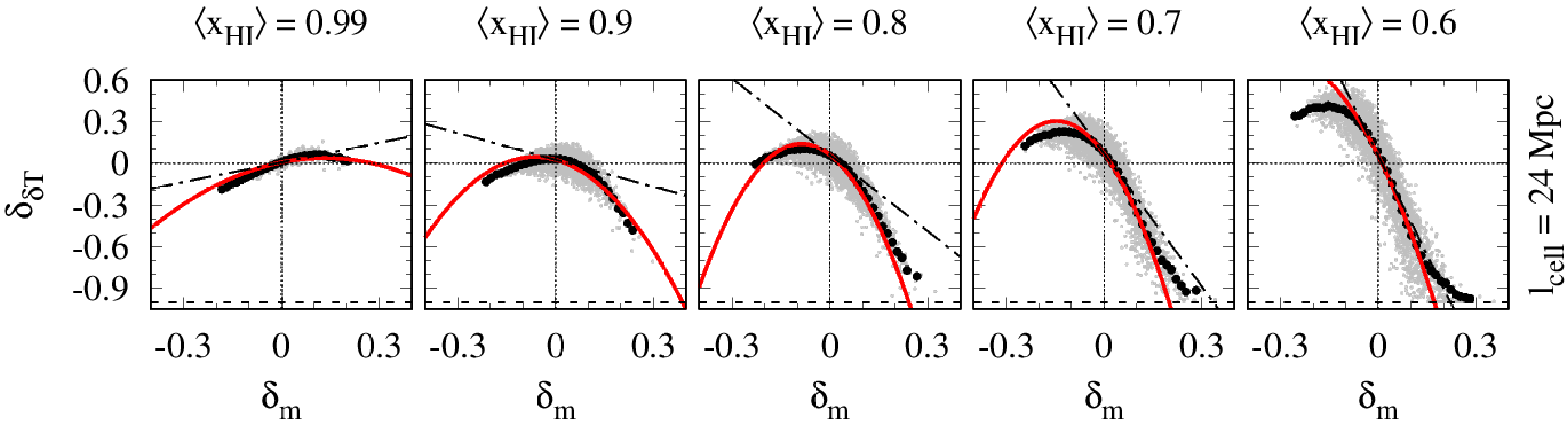
$$\delta_{\delta T} = b_0 + b_1 \delta_m + (b_2/2)\,\delta_m^2$$

$$\left(\langle \delta_{\delta T} \rangle = 0 \rightarrow b_0 = -(b_2/2)\langle \delta_m^2 \rangle \right)$$

bias model validation

quadratic bias model with b1 and b2 from 21cm 3pc:

$$\delta_{\delta T} = b_1 \delta_m + (b_2/2)(\delta_m^2 - \sigma_m^2)$$

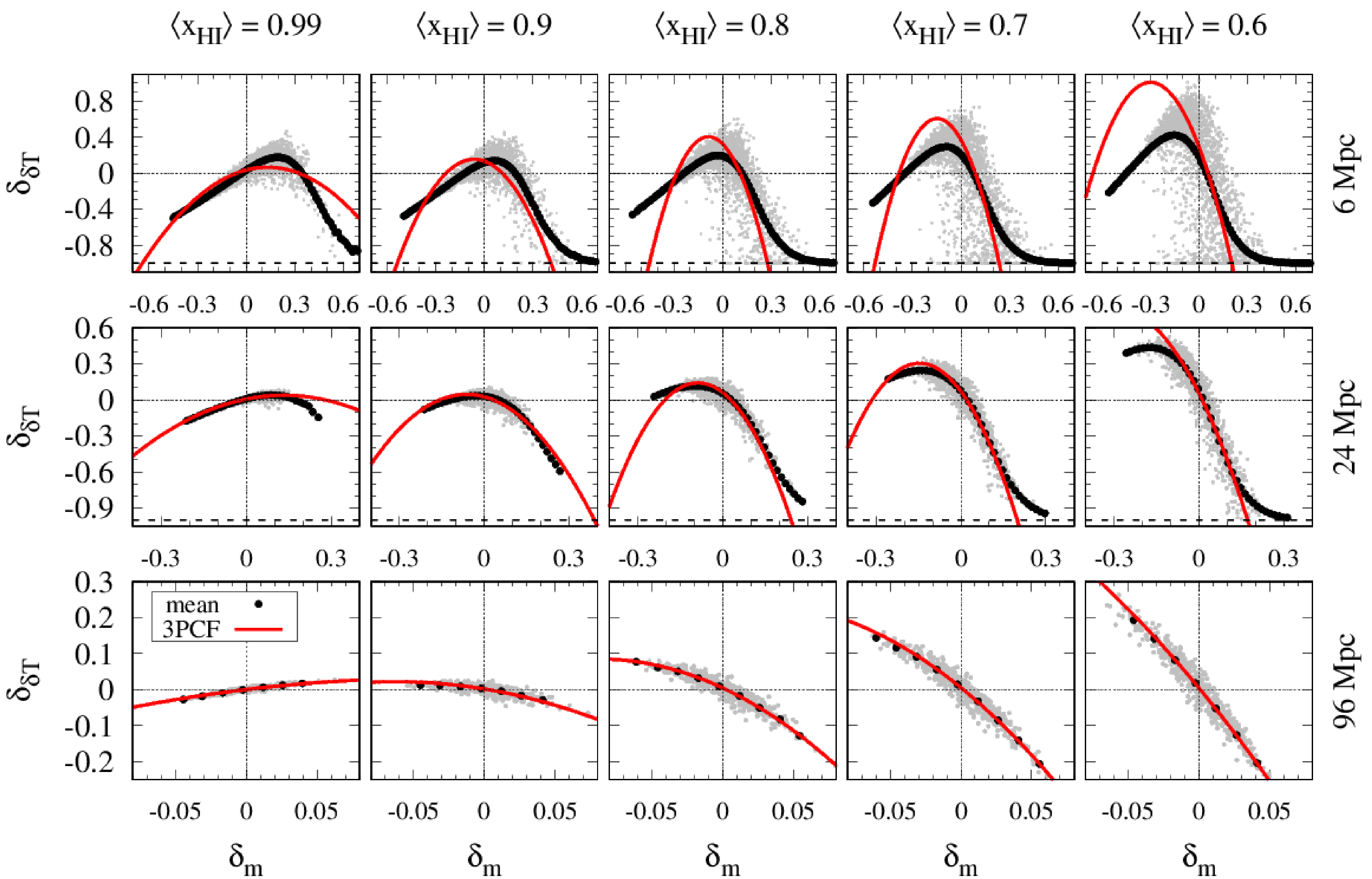


b1 = slope of $\delta_{\delta T}$ **at** $\delta_m = 0$

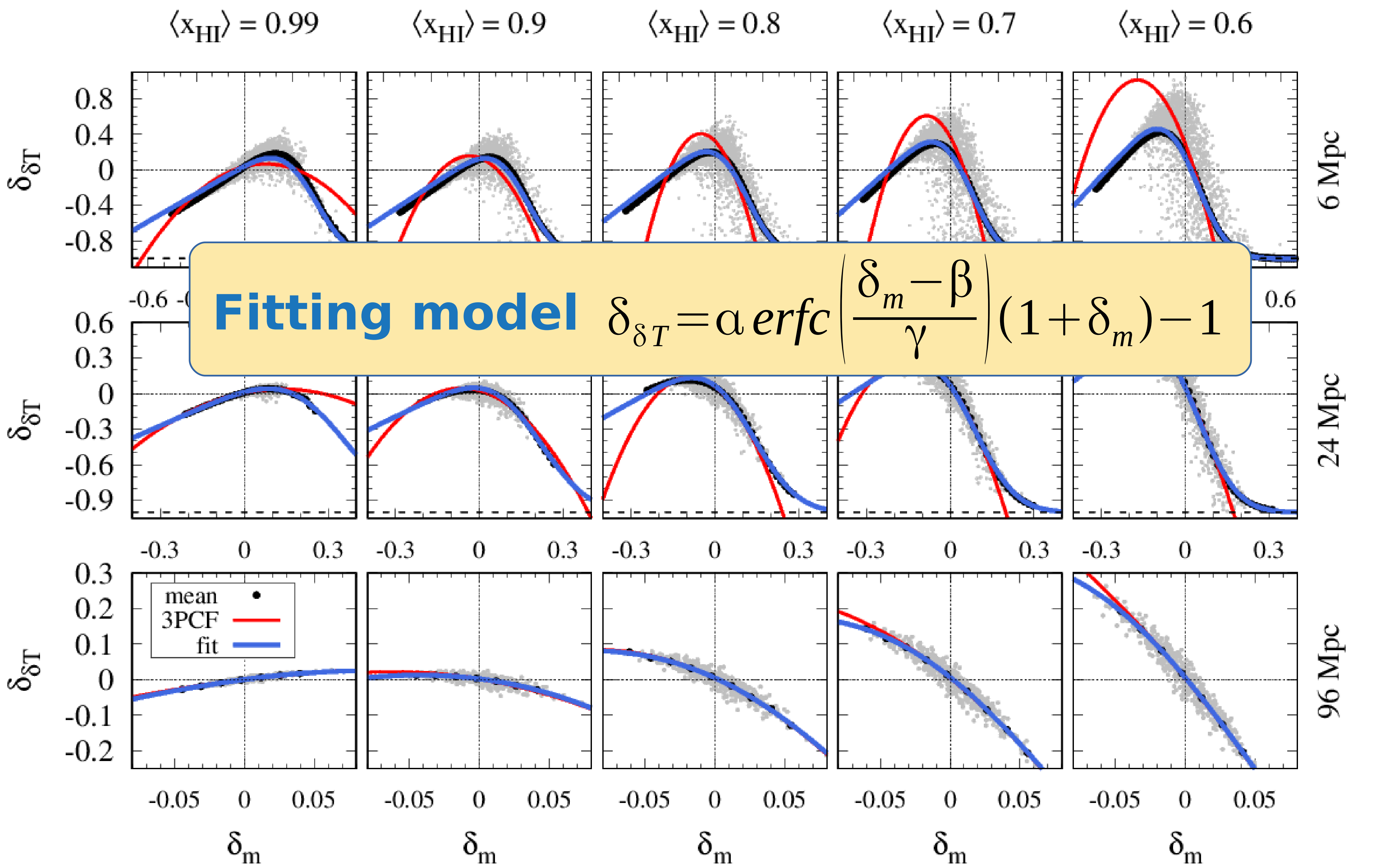
- positive at early times
- negative at late times

$$\delta T \propto x_{HI}(1 + \delta_m)$$

bias model validation



21cm cm bias model



Fitting model $\delta_{\delta T} = \alpha\, erfc\left(\frac{\delta_m - \beta}{\gamma}\right)(1+\delta_m) - 1$

21cm cm bias model

Model for ionized fraction:
Average ionized fraction corresponds to average faction of fluctuations with

$$\delta=\delta_p+\delta_{bg}>B$$

$$\langle X_{HII}\rangle(\delta_{bg})=\frac{1}{\sqrt{(2\pi)}\sigma_p}\int_B^{\infty} e^{-\left(\frac{\delta-\delta_{bg}}{\sqrt{(2)}\sigma_p}\right)^2} d\delta$$

neutral fraction

$$\langle X_{HI}\rangle=1-\langle X_{HII}\rangle=\frac{1}{2}\,erfc\left(\frac{\delta_{bg}-B}{\sigma_p}\right)$$

21cm signal

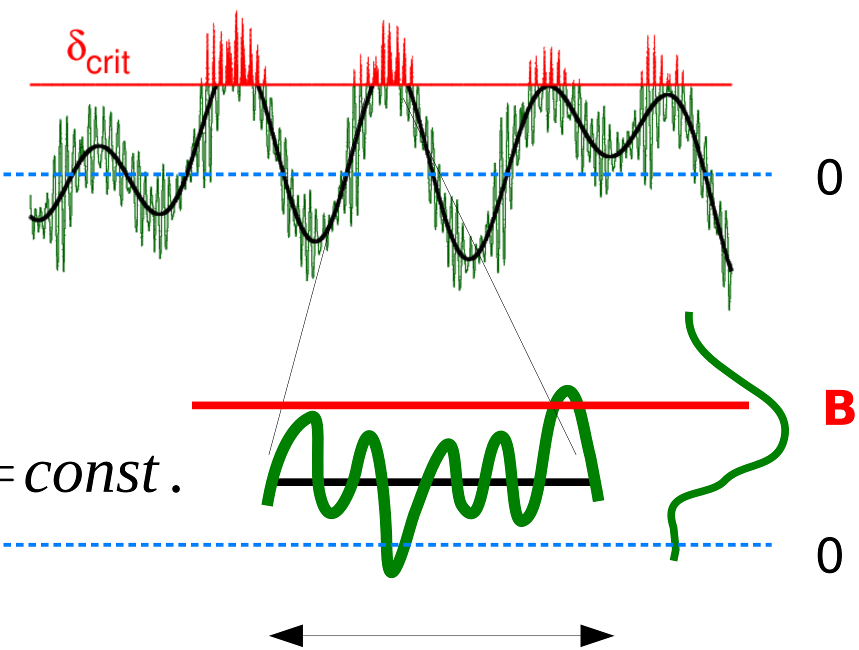
$$\delta T(\delta_m)=\langle X_{HI}\rangle(1+\delta_m)\times const.$$

21cm signal fluctuations

$$\delta_{\delta T}(\delta_m)=\delta T(\delta_m)/\langle\delta T\rangle_{tot}-1$$

$$=> \quad \boxed{\delta_{\delta T}=\alpha\, erfc\left(\frac{\delta_m-B}{\sigma_p}\right)(1+\delta_m)-1}$$

$$\delta=\delta_p+\delta_{bg} \quad => \quad \sigma^2=\sigma_p^2+\sigma_{bg}^2$$



$$\delta_{bg}=const.$$

$$\delta_m=\delta_{bg}$$

Bias parameters

$$b_0=\delta_{\delta T}(0) \quad b_1=\frac{d\delta_{\delta T}}{d\delta_m} \quad b_2=\frac{d^2\delta_{\delta T}}{d\delta_m^2}$$

21cm cm bias model

Model for ionized fraction:
Average ionized fraction co...
to average faction of flu...

$$\delta=\delta_p+\delta_{bg}>B$$

$$\langle X_{HII}\rangle(\delta_{bg})=\frac{1}{\sqrt{(2\pi)}\sigma_p}\int_B^\infty e^{\dots}$$

neutral fraction

$$\langle X_{HI}\rangle=1-\langle X_{HII}\rangle=\frac{1}{2}erfc(\frac{\delta_{bg}-B}{\sigma_p})$$

21cm signal

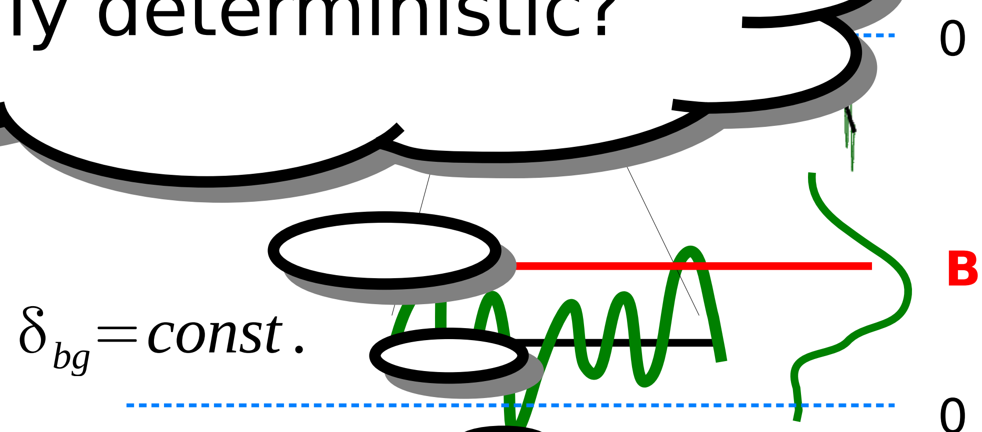
$$\delta T(\delta_m)=\langle X_{HI}\rangle(1+\delta_m)\times const.$$

21cm signal fluctuations

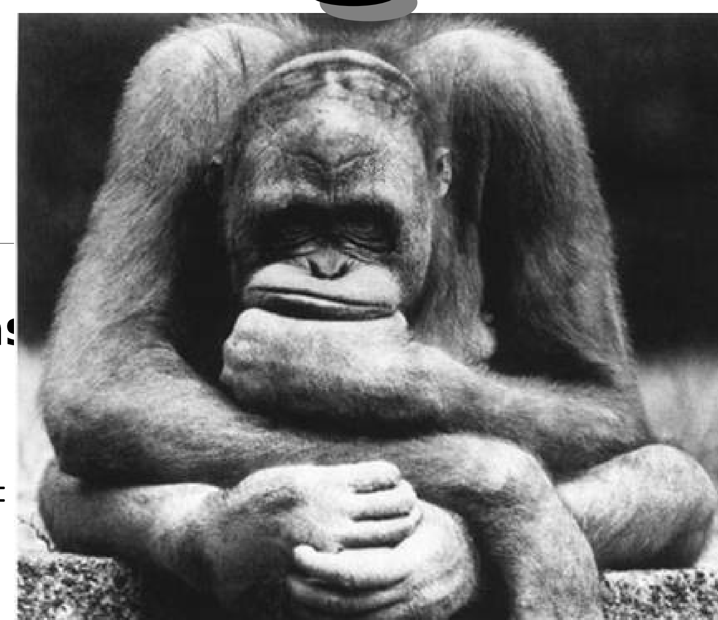
$$\delta_{\delta T}(\delta_m)=\delta T(\delta_m)/\langle\delta T\rangle_{tot}-1$$

=> $$\boxed{\delta_{\delta T}=\alpha\, erfc(\frac{\delta_m-B}{\sigma_p})(1+\delta_m)-1}$$

=> $\sigma^2=\sigma_p^2+\sigma_{bg}^2$

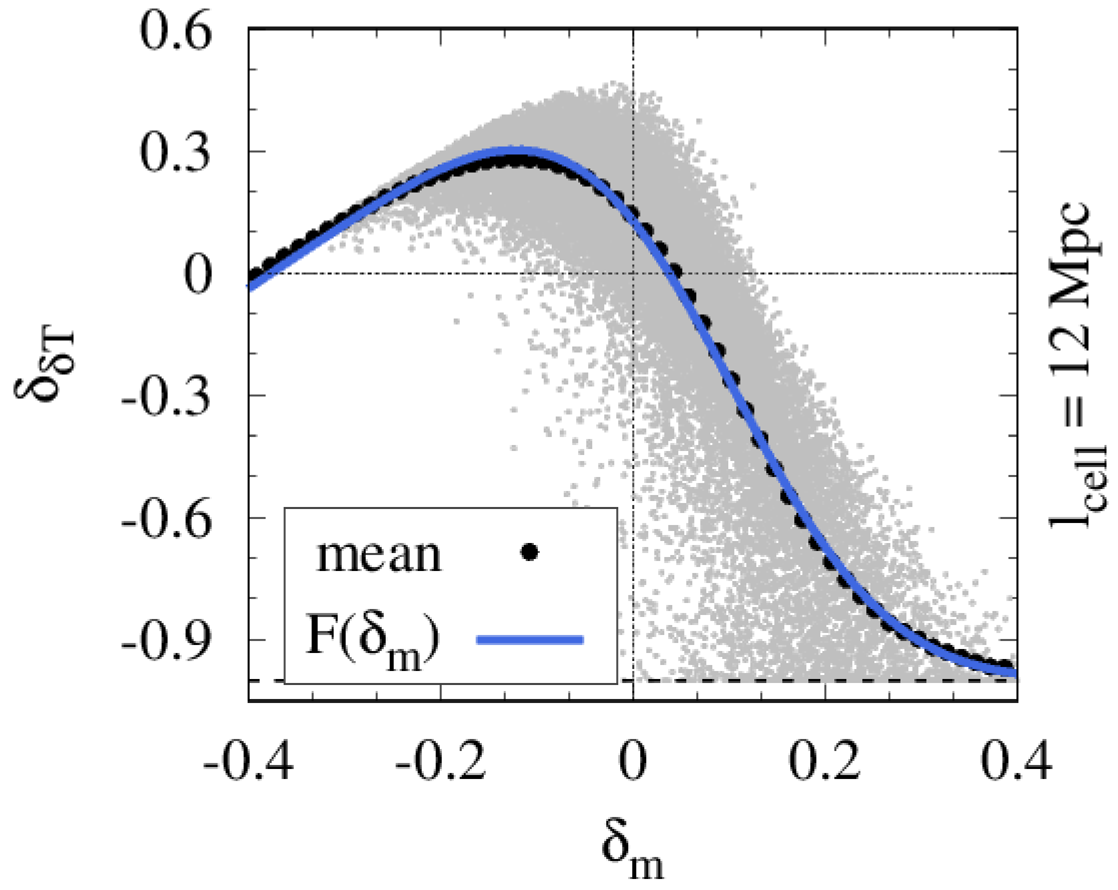


Bia



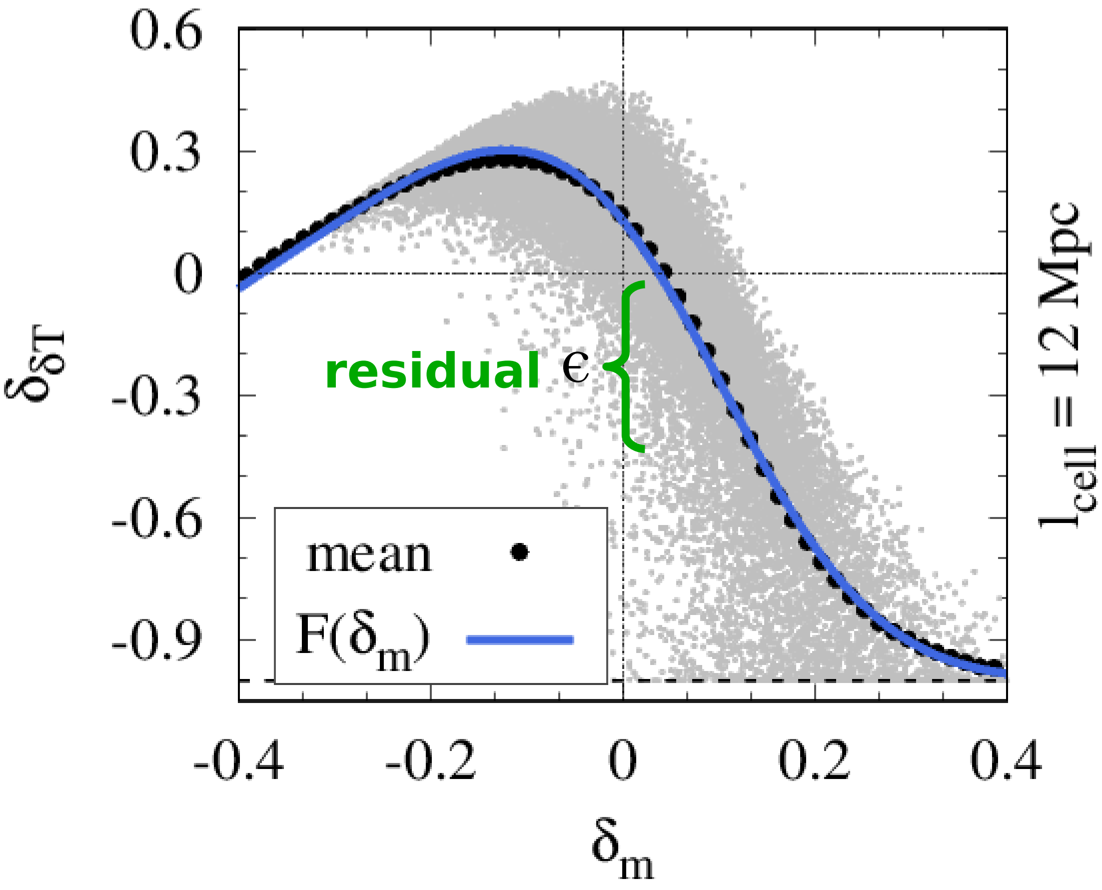
deterministic bias ?

$$\delta_{\delta T} = F(\delta_m)\,?$$

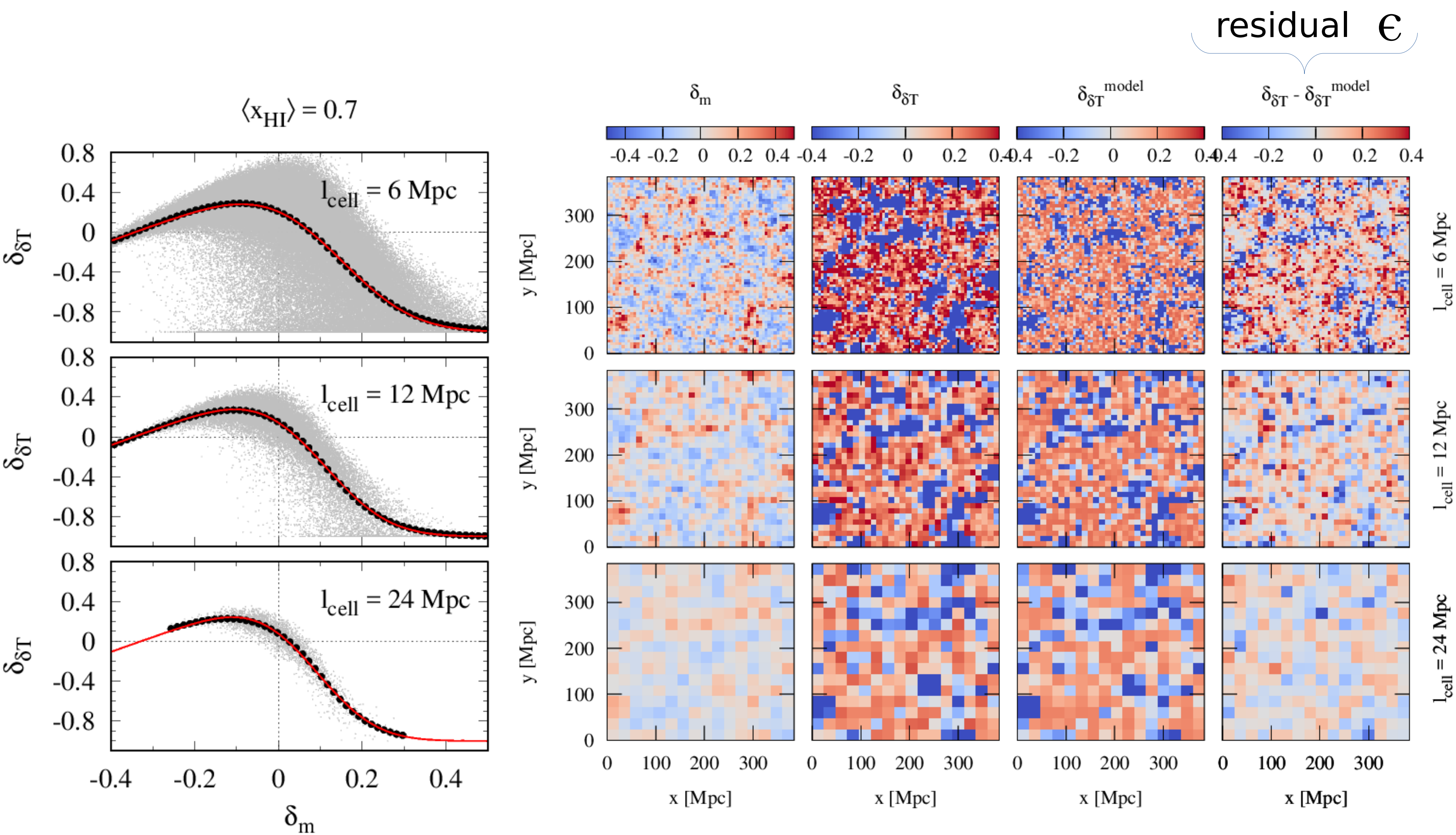


deterministic bias ?

$$\delta_{\delta T} = F(\delta_m) + \epsilon\,?$$



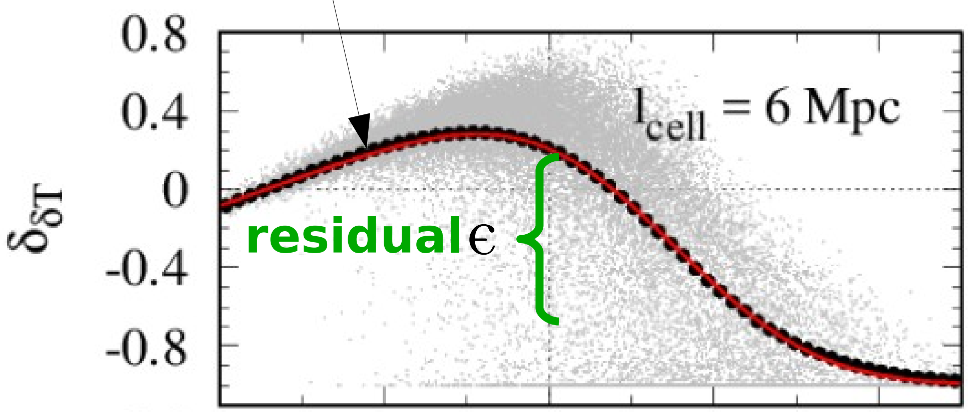
spatial correlation of residuals

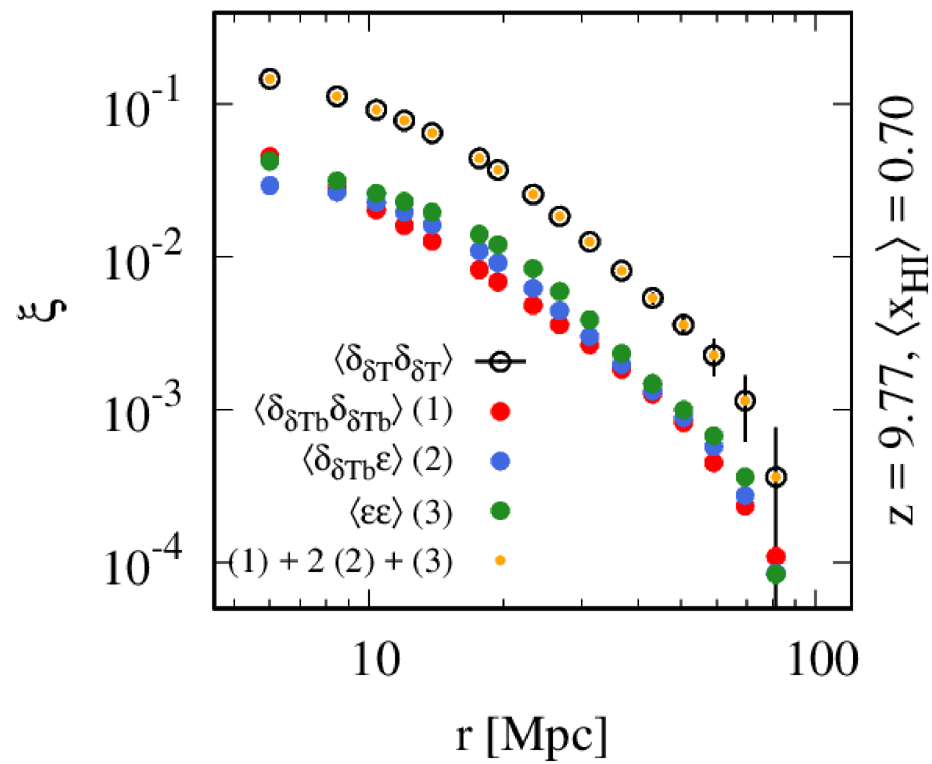


=>bias relation is not deterministic $\delta_{\delta T} = F(\delta_m, ?, \ldots, ?)$

contribution of residuals to 2pc

bias model:





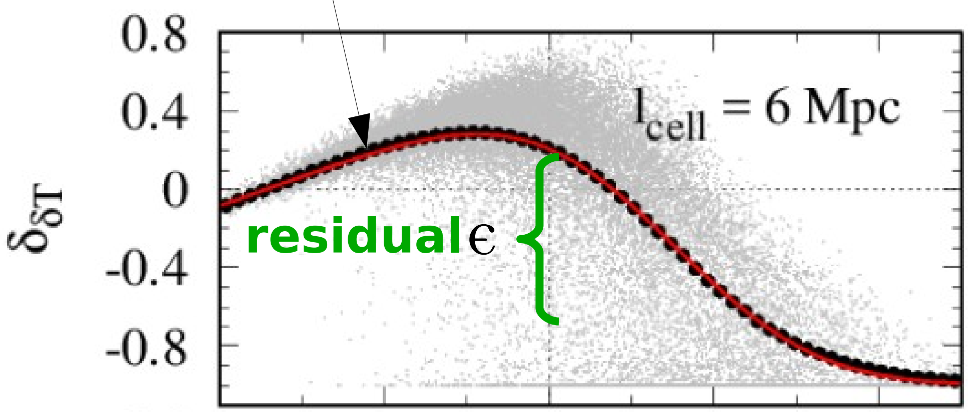
$$\delta_{\delta T} = \delta_{\delta Tb} + \epsilon$$

$$\xi_{\delta T} \stackrel{\text{def}}{=} \langle \delta^{(1)}_{\delta T} \delta^{(2)}_{\delta T} \rangle (r)$$

$$\xi_{\delta T} = \langle \delta^{(1)}_{\delta Tb} \delta^{(2)}_{\delta Tb} \rangle + \underbrace{2 \langle \delta^{(1)}_{\delta Tb} \epsilon^{(2)} \rangle + \langle \epsilon^{(1)} \epsilon^{(2)} \rangle}_{\text{should be zero, if residuals are randomly distributed}}$$

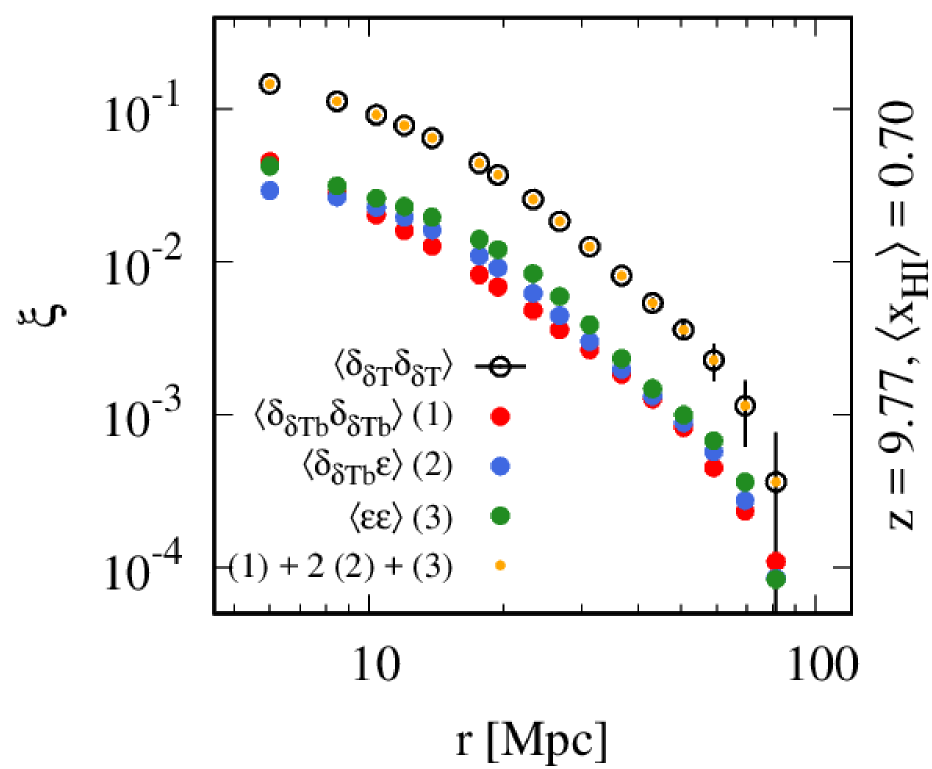
contribution of residuals to 2pc

bias model:



$$\delta_{\delta T} = \delta_{\delta Tb} + \epsilon$$

$$\xi_{\delta T} \stackrel{\text{def}}{=} \langle \delta^{(1)}_{\delta T} \delta^{(2)}_{\delta T} \rangle (r)$$



$$\xi_{\delta T} = \langle \delta^{(1)}_{\delta Tb} \delta^{(2)}_{\delta Tb} \rangle + 2 \langle \delta^{(1)}_{\delta Tb} \epsilon^{(2)} \rangle + \langle \epsilon^{(1)} \epsilon^{(2)} \rangle$$

$$\xi_{\delta T} = b^2_{\delta Tb} \xi_m + 2 b^2_{\delta Tb\epsilon} \xi_m + b^2_{\epsilon} \xi_m = \xi_m b^2_{eff}$$

Cosmology with large-scale 21cm structure

linear growth

$$\delta_m(z) \simeq \delta_m(z_0)\frac{D(z)}{D(z_0)}$$

matter 2pc

$$\xi_m^z \approx \xi_m^{z_0}\left(\frac{D(z)}{D(z_0)}\right)^2$$

growth depends on cosmology

$$D(a) \propto \frac{H(t)}{H(0)}\int_0^a \frac{da'}{[\Omega_m/a' + \Omega_\Lambda a' - (\Omega_m + \Omega_\Lambda - 1)]^{3/2}} \qquad a = \frac{1}{1+z}$$

growth-bias degeneracy $\quad \dfrac{D(z)}{D(z_0)} = \left(\dfrac{\xi_m^z}{\xi_m^{z_0}}\right)^{1/2} = \dfrac{b(z_0)}{b(z)}\left(\dfrac{\xi_{\delta T}^z}{\xi_{\delta T}^{z_0}}\right)^{1/2}$

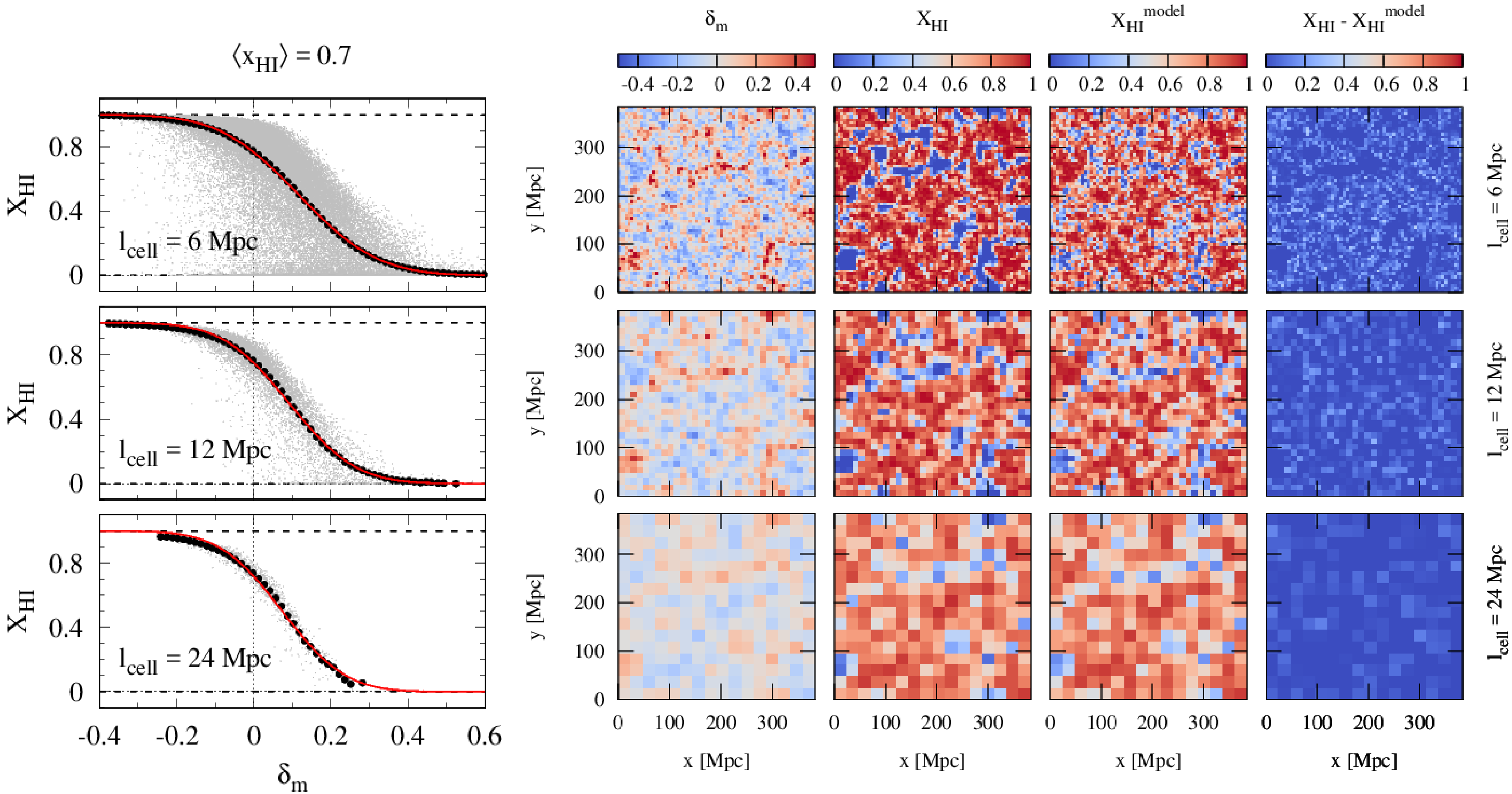
degeneracy broken by combing 2pc & 3PC, if $b_\xi = b_\zeta$

Summary

- global neutral fraction can be predicted without dark matter simulation, using random walks
- 21cm 3pc provides additional info to 21cm 2pc
  → tighter constraints on EoR models
- Quadratic bias model explains shape of 21cm 2pc & 3pc at large scales and early times of reionization (neutral fraction > 60%, r> 20 Mpc)
- b1 from 2pc and 3pc consistent at 10% level
- interpretation of bias parameters not clear due to non-local contributions to the bias model
- possible application of bias model for measuring cosmological parameters during EoR

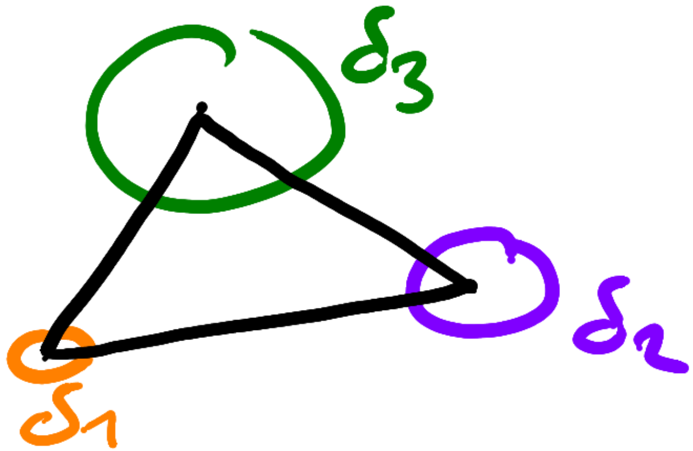
spatial correlation of residuals

Model: $$X_{HI}(\delta_m) = a\,erfc(b\delta_m + c)$$

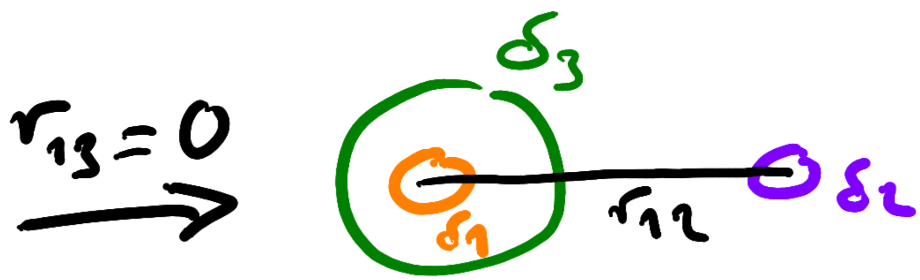


conditional large scale fluctuations for two fixed small scale fluctuations

general case



case for computing large scale matter density delta 3 around delta 1, given delta 2



expectation value and variance for delta3 for fixed delta 1 and delta 2 from 3D normal distribution:

$$\bar{\delta}_3=\frac{\delta_1(\hat{\xi}_{13}-\hat{\xi}_{23}\hat{\xi}_{12})+\delta_2(\hat{\xi}_{23}-\hat{\xi}_{13}\hat{\xi}_{12})}{1-\hat{\xi}_{12}^2}$$

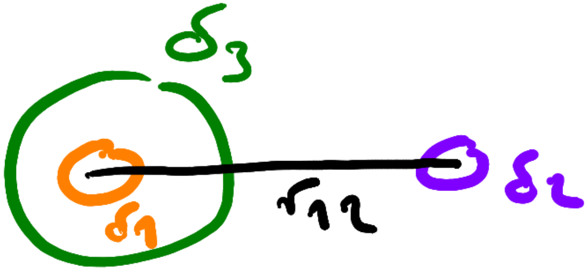
$$\hat{\xi}_{ij}=\xi_{ij}/\sigma^2$$

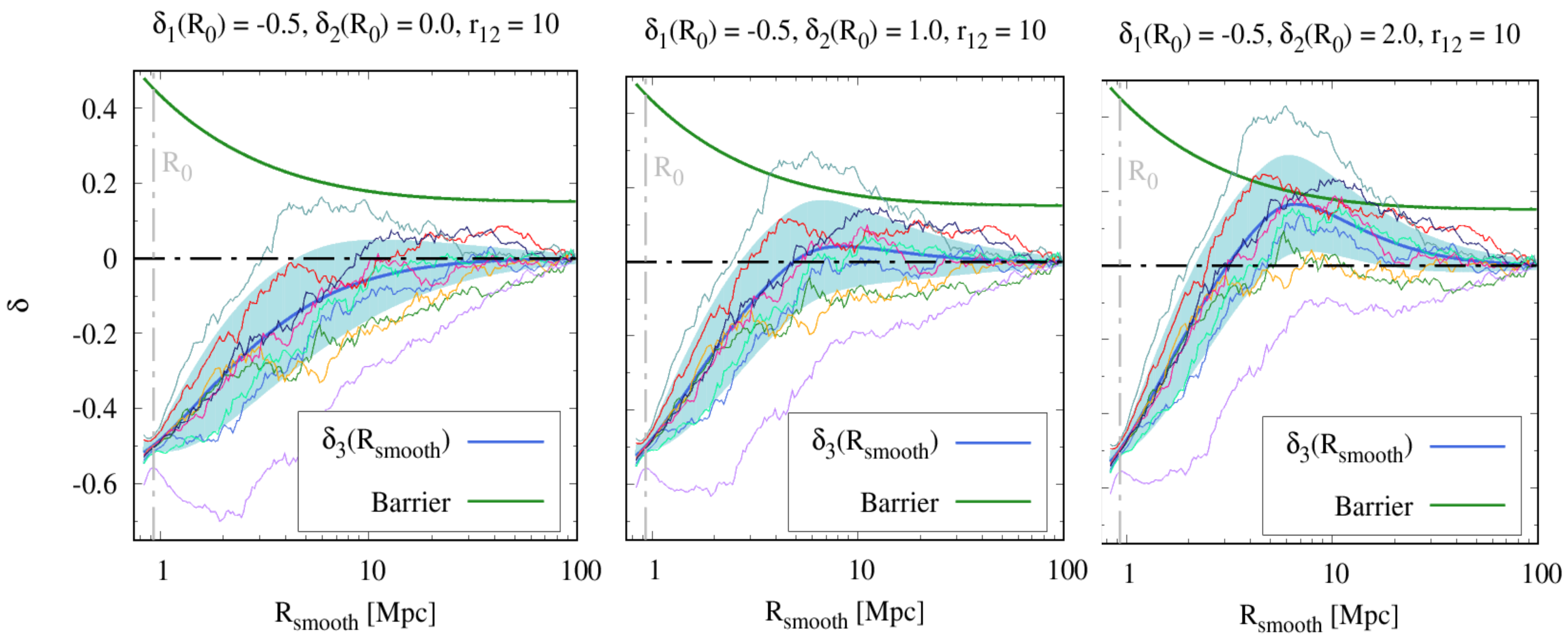
$$\bar{\sigma}_3^2=\sigma_3^2-\frac{\xi_{13}(\hat{\xi}_{13}-\hat{\xi}_{23}\hat{\xi}_{12})+\xi_{23}(\hat{\xi}_{23}-\hat{\xi}_{13}\hat{\xi}_{12})}{1-\hat{\xi}_{12}^2}$$

$$\sigma^2=\langle\delta_1^2\rangle=\langle\delta_2^2\rangle$$

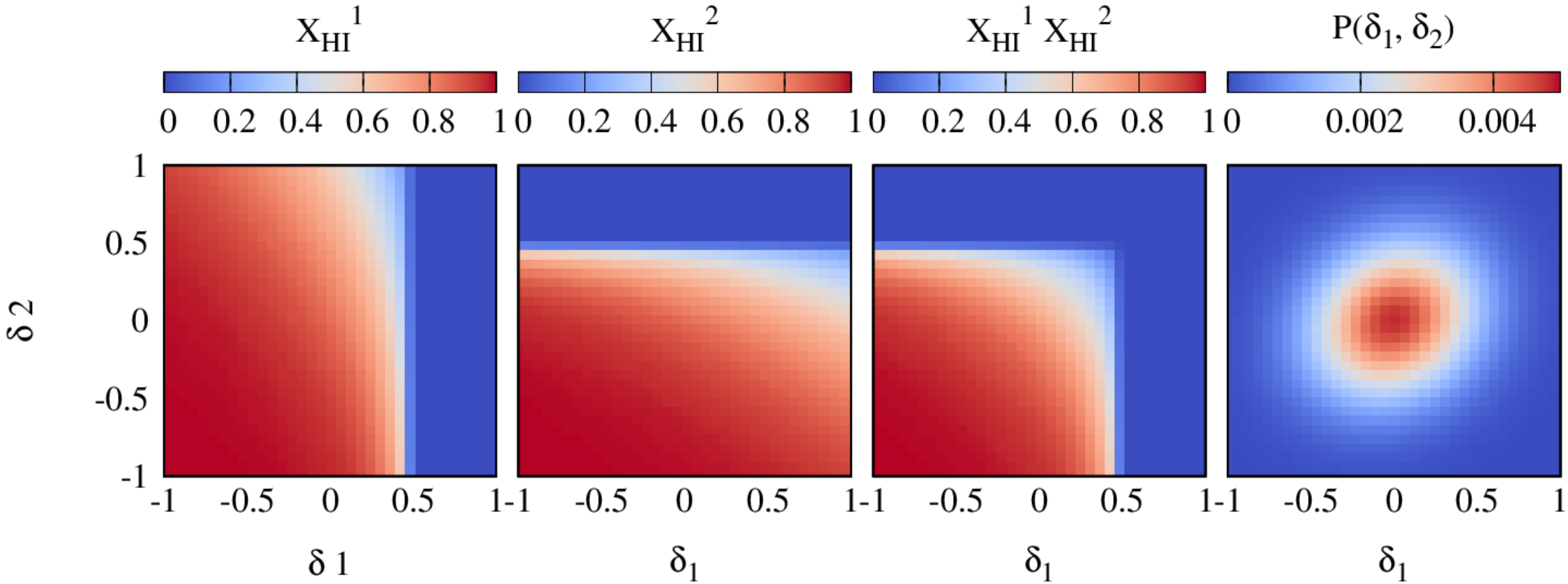
trajectories of smoothed matter density

trajectories of smoothed fluctuations delta 3 around delta 1 for fixed values of delta 1 and delta 2





$X_1(\delta_1,\delta_2)$ from conditional random walks

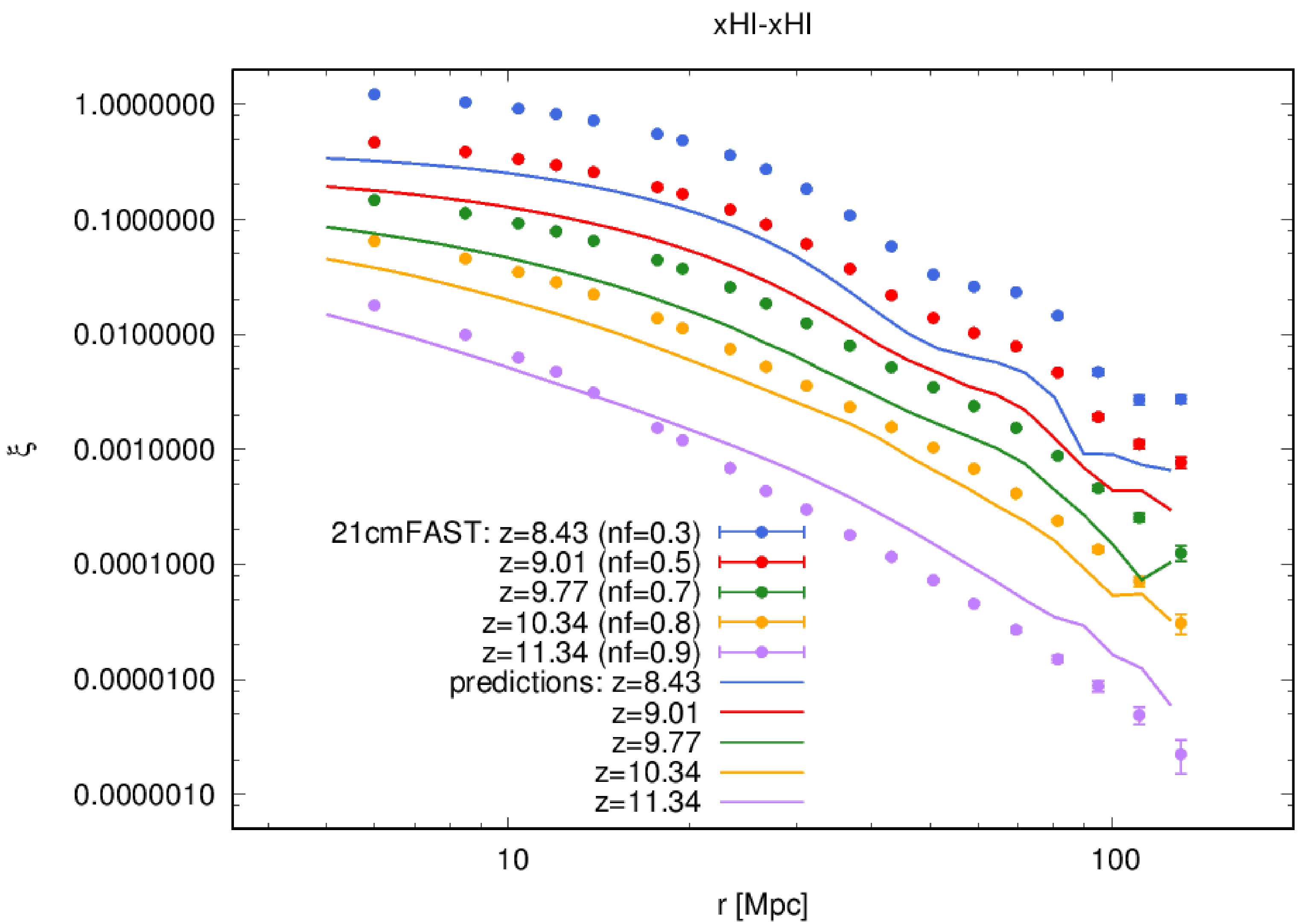


2pc model: $$\langle X_1 X_2\rangle = \iint_{-1}^{\infty} P(\delta_1,\delta_2)\langle X_1 X_2\rangle(\delta_1,\delta_2)\, d\delta_1\, d\delta_2$$

assuming: $$\langle X_1 X_2\rangle(\delta_1,\delta_2) = \langle X_1\rangle(\delta_1,\delta_2)\langle X_2\rangle(\delta_1,\delta_2)$$

and using: $$\langle X_2\rangle(\delta_1,\delta_2) = \langle X_1\rangle(\delta_2,\delta_1)$$

predictions for 2pc of neutral fraction



problems with normalization, probably because of wrong variance for delta3(delta1, delta2))

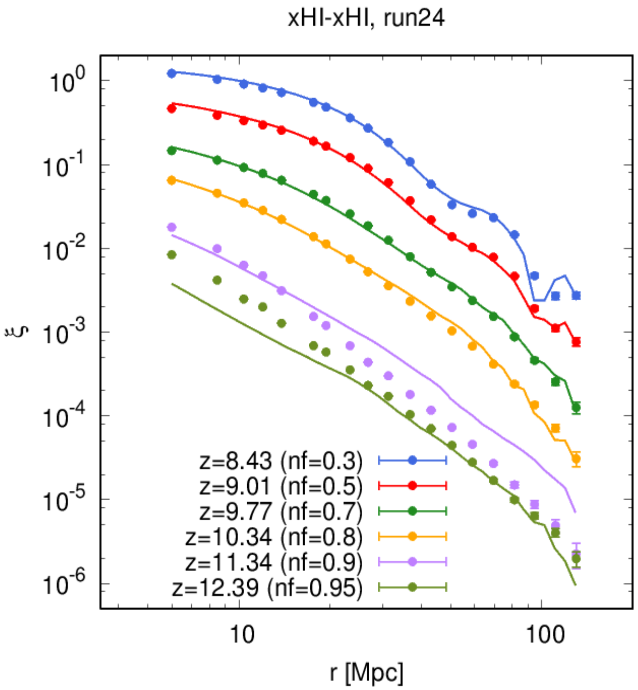
predictions for 2pc of neutral fraction

adding empirical correction factor to predictions:

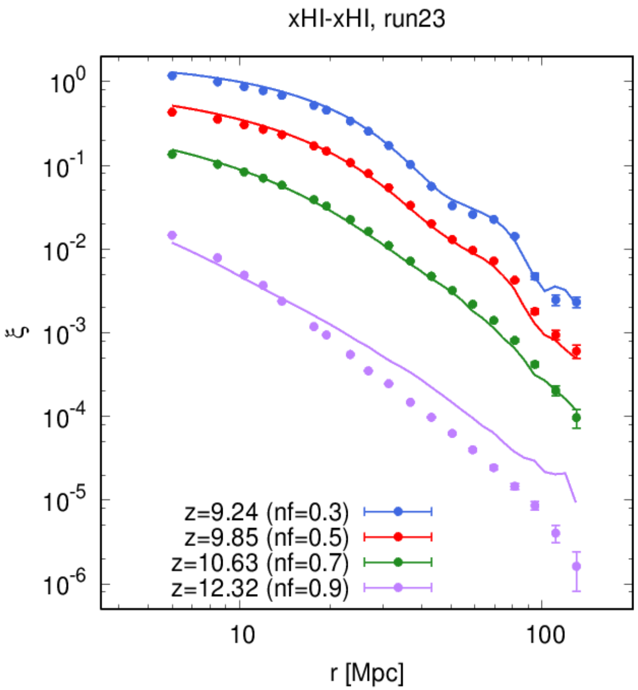
$$\xi_{corr} = \xi f_{corr}(\langle X \rangle) \qquad f_{corr}(\langle X \rangle) = 5.5(1.1 - \langle X \rangle)$$

=> corrected predictions agree with measurements for different EOR models with different parameters at low z

model A



model B



model C